MATH 4650/CS 4656 - Numerical Analysis I
Worksheet Chapter 3

2.1) For $f(x) = x^2 - 2 = 0$, in the interval $[1, 2]$, $(a = 1, b = 2)$, use the bisection method to compute the first 3 iterates and associated errors. Verify that the inequalities hold. What is the error at the n^{th} iterations? That is, $e_n =$.

$x_1 =$
$x_2 =$
$x_3 =$

2.2) Carry out 4 iterations of Newton's method for $f(x) = x^2 + 3x - 4 = 0$, (solution is : $\{x = 1\}, \{x = -4\}$ of course) for

$x_0 = 0$	2	-3	-1.5	-5
$x_1 =$	$x_1 =$	$x_1 =$	$x_1 =$	$x_1 =$
$x_2 =$	$x_2 =$	$x_2 =$	$x_2 =$	$x_2 =$
$x_3 =$	$x_3 =$	$x_3 =$	$x_3 =$	$x_3 =$
$x_4 =$	$x_4 =$	$x_4 =$	$x_4 =$	$x_4 =$
$x_5 =$	$x_5 =$	$x_5 =$	$x_5 =$	$x_5 =$

Do you see quadratic convergence?

2.3) The "old" Cray supercomputers does/did not have a divide unit. Instead, to compute 1/R, R > 0, it performs a "reciprocal approximation" accurate to about half a floating point word, then uses it as an initial guess for one Newton iteration where $f(x) = 1/x - R = 0$.

a) Find 1/R, R = 1, 2, ..., 10 where $x_0 = 0.01$. Carry out 4 iterations. What is the error? How many iterations would be required to get 8 digits of accuracy when we use this initial point?

b)Use the Newton's method on the "reciprocal approximation" to find four iterations of 1/7 with your initial guess for 1/R.

$x_0 =$
$x_1 =$
$x_2 =$
$x_3 =$
$x_4 =$

2.4)For $f(x) = x^2 + 3x - 4 = 0$, carry out 5 iterations of secant method for

x_0 and x_1	3 and 2	-6 and -5	0 and -1	0 and 0.5
$x_2 =$				
$x_3 =$				
$x_4 =$				
$x_5 =$				
$x_6 =$				

2.5) Fixed point iteration: For $f(x) = x^2 - 2 = 0$, carry out 4 iterations of the fixed point iteration, using the starting points of $x_0 = 1, 2$ and where:

a) $F(x) = x^2 + x - 2$

$x_0 = 1$
$x_1 =$
$x_2 =$
$x_3 =$
$x_4 =$
$x_0 = 2$
$x_1 =$
$x_2 =$
$x_3 =$
$x_4 =$

b) $\alpha f(x) = 0$, and $\alpha = 0.25, -0.25$ **Note** that this makes $F(x) = \alpha x^2 + x - 2\alpha$

$x_0 = 1$
$x_1 =$
$x_2 =$
$x_3 =$
$x_4 =$
$x_0 = 2$
$x_1 =$
$x_2 =$
$x_3 =$
$x_4 =$