

MATH 4-5794: Optimization Modeling - Spring  
Semester 2005  
Assignment 3

**Date Due:**

Note: The word solve means, "Use MATLAB (or Excel or GAMS or Lindo or another mathematical programming software) to find the solution. What you turn in will be the source followed by the solution (in sequence corresponding to the problems given below).

**Undergraduates** (8 points total)

1. Solve the problem on page 145 for integer variable:
  - (a) Do two full iterations by hand using the branch and bound algorithm.
  - (b) Using a computer system.
2. Solve the set covering problem on page 176.
3. Solve the bin packing problem on page 178.
4. Solve the following knapsack problem,

$$\begin{aligned} & \max 60x_1 + 60x_2 + 40x_3 + 10x_4 + 20x_5 + 10x_6 + 3x_7 \\ \text{subject to} & \quad 3x_1 + 5x_2 + 4x_3 + x_4 + 4x_5 + 3x_6 + x_7 \leq 10 \\ & \quad x_j \in \{0, 1\} \end{aligned}$$

- (a) Do two full iterations by hand using the branch and bound algorithm.
  - (b) Using a computer system.
5. Solve example 1 on page 210.
6. Solve example 3 on page 212.

**Graduates** (5 points total)

1. Consider all optimization models that our author presents in chapters 1-9. Develop a classification system for these models and put the models into your classification scheme. For example, the knapsack problem would fit under my Integer Programming classification.
2. Solve the following problem,

$$\begin{aligned} & \max 3x_1 + 3x_2 + 13x_3 \\ \text{subject to} & \quad : \\ -3x_1 + 6x_2 + 7x_3 & \leq 8 \\ 6x_1 - 3x_2 + 7x_3 & \leq 8 \\ 0 & \leq x_j \leq 5 \text{ and an integer} \end{aligned}$$

- (a) Do two full iterations by hand using the branch and bound algorithm.
  - (b) Using a computer system.
3. Set up and solve (slight modification from *Introduction to Operations Research* by F. S. Hillier and G. J. Lieberman): Frontier Airlines is considering the purchase of new long-, medium-, and short-range jet passenger planes. The purchase price would be \$33,500,000 for each long-range plane, \$25,000,00.00 for each medium-range plane, and \$17,500,000 for each short-range plane. The board of directors has authorized a maximum commitment of \$750,000,000 for these purchases. Regardless of which airplanes are purchased, air travel of all distances is expected to be sufficiently large enough so that these planes would be utilized at essentially maximum capacity. It is estimated that the net annual profit (after subtracting capital recovery costs) would be \$2,100,000 per long-range plane, \$1,500,00 per medium-range plane, and \$1,150,000 per short-range plane. It is predicted that enough trained pilots will be available to the company to crew 30 new airplanes. If only short-range planes were purchased, the maintenance facilities would be able to handle 40 new planes. However, if each medium-range plane is equivalent to  $1\frac{1}{3}$  short-range planes, and each long-range plane is equivalent to  $1\frac{2}{3}$  short-range planes in terms of their use of the maintenance facilities. The information above was obtained by a preliminary analysis of the problem. A more detailed analysis will be conducted later by Frontier. However, using the preceding data as a first approximation, the management wishes to know how many planes of each type should be purchased to maximize profit. That is, solve the above with a maximization of profit objective function.
- (a) Formulate the Integer Programming model for this problem.
  - (b) Solve the problem on the computer.