

**SOLUTION (Find the Integral)**

$$\begin{aligned}\int_0^1 x f''''(2x) dx &= \frac{1}{2} \int_0^1 x \frac{d}{dx} [f''(2x)] dx \\ &= \frac{1}{2} [x f''(2x)]_0^1 - \int_0^1 f''(2x) dx \\ &= \frac{1}{2} \left[ 5 - \frac{1}{2} f'(2x) \right]_0^1 \\ &= \frac{1}{2} \left[ 5 - \frac{1}{2} f'(2) + \frac{1}{2} f'(0) \right] \\ &= 2\end{aligned}$$