

Solution (Patterns and a Formula)

The following pattern gives the rest of the triangular numbers.

$$\begin{aligned}0 + 2 &= 3 \\0 + 5 + 5 &= 10 \\0 + 7 + 7 + 7 &= 21 \\0 + 9 + 9 + 9 + 9 &= 36 \\&\cdot \\&\cdot \\&\cdot\end{aligned}$$

The pattern for odd triangular numbers generalizes to

$$a_n = 1 + (n+2)\frac{(n+1)}{2} \text{ for odd } n.$$

The pattern for even triangular numbers generalizes to

$$a_n = \frac{n}{2}(n+1) \text{ for even } n.$$

The formula for the n th triangular number is

$$a_n = \frac{n(n+1)}{2} \text{ a familiar formula.}$$