What is a proof?

Proofing as a social process, a communication art.

Theoretically, a proof of a mathematical statement is no different than a logically valid argument starting with some premises and ending with the statement you want proved. However, in the real world such logically valid arguments can get so long and involved that they lose their "punch" and require too much time to verify.

What is a proof?

In mathematics, the purpose of a proof is to *convince* the reader of the proof that there is a logically valid argument in the background. Both the writer and the reader must be convinced that such an argument can be produced – if needed.

For someone to say that they “can't do proofs” is tantamount to admitting that they can not convince anyone that they can think logically.

What is a Proof?

Writing mathematical proofs is therefore an art form (the art of convincing) and a social process since it is directed at people (the readers).

A mathematical proof of a statement strongly depends on *who* the proof is written for. Proofs for a research audience are quite different from those found in textbooks. And even textbook proofs look different depending on the level of the audience (high school vs. college vs. graduate school).

To simplify our task in this course, you will write all of your proofs with a specific audience in mind:

ME!

What is a Proof?

That is, you are writing to convince me that you could drop down to the logic level and provide all the details, if I asked you to do so.

Rigor in proofs.

The above remarks should not be construed to mean that you can get sloppy with your proofs – your audience requires clarity, precision and, above all, correctness.

Phrases such as "clearly" or "it is easy to see that" are neither clear nor easy for this audience.

When you say something follows from a definition, I want to know "the definition of what?"

General Hints

The importance of definitions.

It can not be overemphasized how important definitions are. Without a clear and crisp understanding of a definition, you will not be able to use it in a proof. You have to be able to recall a definition precisely when it is needed – vague familiarity will not work for you.

General Hints

Working backwards.

There is a big difference between discovering a proof and presenting a proof. In presenting a proof you must be convincing, and things need to follow in a logical order. To discover a proof, you are under no such restrictions and often the best procedure is to work the problem backwards.

General Hints

Mathematics: the Language

Written and spoken mathematics is a very precise language (which is only a subset of the general language). Students run into difficulty when they do not understand this point. Reading too much into a mathematical statement or not getting the full implication out of a statement are the main problems students have with mathematics. Proofs require that you use this language correctly. (If you can not use the language correctly, you will have a hard time convincing me that you know what you are talking about!)