

Math 3210

Sample Final Exam

Part I (40 points) - You must answer the four questions in this part. Each question is worth 10 points.

1. Determine whether the following statements are true or false:

- a. **T** If two triangles are congruent, they have the same defect.
- b. **T** The Euclidean parallel postulate is equivalent to the existence of one rectangle.
- c. **F** The line joining the midpoints of the equal sides of a Saccheri quadrilateral is perpendicular to both sides.
- d. **F** If two triangles are congruent in the Poincaré model, then they are congruent in the Euclidean sense.
- e. **T** In hyperbolic geometry, every angle has a unique bisector.

2. The centuries long search for a proof of Euclid's parallel postulate was doomed to failure. Write a short paragraph explaining why.

3. In the hyperbolic plane, show that if two lines are cut by a transversal so that the alternate interior angles are equal, then the lines do not intersect.

4. a) In the Poincaré model, draw an example of an omega isosceles triangle one of whose vertices is at the center of the model.

b) In hyperbolic geometry, given an acute angle, it is possible to have a line perpendicular to one side of the angle and parallel to the other side. In the Poincaré model, draw an example of this phenomenon.

Part II (80 points) - Answer any four (4) questions from this section. Clearly indicate which questions you have answered. Each question is worth 20 points.

5. Prove that the radius of the 9-point circle of a triangle is half the radius of the circumcircle of that triangle.

6. Define the following *italicized* terms by completing the sentences below. (Your definitions need not be verbatim reproductions of the book or class notes, but they must be correct!)

a) An *excenter* of a triangle is ...

b) The *Euler line* of a triangle is the line segment determined by ...

c) A *median* of a triangle is ...

- d) A *constructible number* is the length of a line segment which ...
- e) The *9-point circle* of a triangle contains the following 9 points of a triangle :
- f) The *defect* of a triangle is
- g) Given a point P and a line l not passing through P, the *angle of parallelism* of l at P is ...
- h) A *Lambert quadrilateral* is a
- i) The *summit* of a Saccheri quadrilateral is
- j) An *omega (ideal)* point in hyperbolic geometry is ...

7. In *Young's geometry*, whose axioms are:

- a. There exists at least one line.
- b. Every line of the geometry has exactly three points on it.
- c. Not all the points are on the same line.
- d. For each two distinct points, there exists exactly one line on both of them.
- e. If a point does not lie on a given line, then there exists exactly one line on that point that does not intersect the given line (Playfair's Axiom).

Prove that two lines parallel to a third line are parallel to each other.

8. (a) Write the equations for the glide reflection which consists of a reflection about the x-axis followed by a translation by the vector $(5, 0)$.
 (b) Find the inverse of this glide reflection and write its equations.

9. Answer these 5 questions:

- a. Could the centroid be outside the triangle?
- b. Could the incenter be outside the triangle?
- c. Could the circumcenter be outside the triangle?
- d. Where is the orthocenter of a right triangle?
- e. In what type of triangle could the incenter, circumcenter, centroid and orthocenter be different points, yet all lie on the same line?

10. In any Saccheri quadrilateral, show that the segment joining the midpoints of the base and summit, and the segment joining the midpoints of the two equal sides are perpendicular.

11. *Fano's Geometry* can be described by the following axiom set (slightly different from the one in the text):

1. Every line of the geometry has exactly three points on it.
2. Every pair of distinct points is on exactly one line.
3. Every pair of distinct lines have exactly one point in common.
4. There exists a set of four points, no three of which are on the same line.

Show that for any set of three points which are not on the same line, there is a **unique** fourth point so that these four points satisfy axiom 4. (*You may use, without reproving, any theorem concerning Fano's geometry that was talked about in class*).

12. Prove that the medians of any triangle meet at a point (are concurrent).

13. Write a paragraph which expresses your understanding of the angle trisection problem. Include in this paragraph specific statements dealing with the following points.

- a. Give an accurate statement of the problem.
- b. What is the mathematical status of the problem?
- c. How important is this problem in the area of geometric constructions?
- d. How would you respond to someone who insisted that they had an angle trisection construction?