Math 3000 Homework #1 Answers

pg. 40 # 6

f is discontinuous at a point a if

$(\exists\varepsilon > 0)(\forall\delta > 0)(\exists x)(| x - a | < \delta \ \wedge \ | f(x) - f(a) | \geq \varepsilon)$

pg. 49 # 6

Let x be an odd integer > 5. Then x = 2n + 1 with n > 2. Now, x = 2(n-1) + 3 and since 2(n-1) is an even integer > 2, by the assumption there exist primes p and q so that 2(n-1) = p + q. Therefore, x = p + q + 3 and is the sum of three primes.

pg. 60 # 9

Assume $x \in A$. Since $A = \{x \mid P(x)\}$ we have that P(x) is true. Since we are given that $(\forall x)\ (P(x) \Rightarrow Q(x))$ we also have that Q(x) is true. Since $B = \{y \mid Q(y)\}$, we have that $x \in B$. Thus, $(\forall z)(z \in A \Rightarrow z \in B)$, so $A \subseteq B$. In order to show that $A \subset B$, we would have to produce an $x \in B$ which is not in A, i.e., an x for which P(x) is false and Q(x) is true. The existence of such an x is certainly possible, but there is not enough information given to show that such an x must exist. For instance, if both P(x) and Q(x) are tautologies, the condition is satisfied and A = B. Thus, $A \subset B$ can not be proved.

pg. 67 # 10c

Assume $x \in A$. Since $A \subseteq B \cup C$, $x \in B \cup C$. Thus, either $x \in B$ or $x \in C$. Since $A \cap B = \emptyset$, we have $x \notin B$ and so we must have $x \in C$. Therefore, $A \subseteq C$.

pg. 78 # 5

Let $\mathscr{F}$ be a family of sets. $\cap\mathscr{F}$ is the largest set X such that $X \subseteq A$ for all $A \in \mathscr{F}$. Proof: By Theorem 3.6.1, $\cap\mathscr{F} \subseteq A$ for all $A \in \mathscr{F}$. If X is a larger set with this property, then there exists an $x \in X$ with $x \notin \cap\mathscr{F}$. But, if $x \notin \cap\mathscr{F}$ then $\exists\ A \in \mathscr{F}$ with $x \notin A$, so we would have $X \not\subseteq A$ for this A, contradicting the definition of X. Therefore, $X = \cap\mathscr{F}$.