Additional Homework Problems Math 3000 Fall 2006

2. Suppose $|A| = 24$, $|B| = 21$, $|A \cup B| = 37$, $|A \cap C| = 11$, $|B - C| = 10$ and $|C - B| = 12$. Find
   a) $|A \cap B|$ b) $|A-B|$
   c) $|B - A|$ d) $|B \cup C|$
   e) $|C|$ f) $|A \cup C|$

4. Of the four teams in a softball league, one team has four pitchers and the other teams have three each. Give the counting rules that apply to determine each of the following.
   (a) The number of possible selections of pitchers for an all-star team, if exactly four pitchers are to be chosen.
   (b) The number of possible selections if one pitcher is to be chosen from each team.
   (c) The number of possible selections of four pitchers, if exactly two of the five left-handed pitchers in the league must be selected.
   (d)The number of possible orders in which the four pitchers, once they are selected, can appear (one at a time) in the all-star game.

6. (a) If you have 10 left shoes and 9 right shoes and do not care whether they match, how many "pairs" of shoes can you select?
   (b) A cafeteria has 3 meat selections, 2 vegetable selections, and 4 dessert selections for a given meal. If a meal consists of one meat, one vegetable and one dessert, how many different meals could be constructed?
   (c)There are 3 roads from Abbottville to Bakerstown, 4 roads from Bakerstown to Cadez, and 5 roads from Cadez to Detour Village. How many different routes are there from Abbottville through Bakerstown and then Cadez to Detour Village?

10. Find the number of ways seven school children can line up to board a school bus.

11. Suppose the seven children of exercise 10 (above) are three girls and four boys. Find the number of ways they could line up subject to these conditions.
   (a) The three girls are first in line.
   (b) The three girls are together in line.
   (c) The four boys are together in line.
   (d) No two boys are together.

14. From a second-grade class of 11 boys and 8 girls, 3 are selected for flag duty.
   (a) How many selections are possible?
   (b) How many of these selections have exactly two boys?
   (c) Exactly one boy?

17. Find
   (a) $(a + b)^6$
   (b) $(a + 2b)^4$
   (c) the coefficient of a^3b^{10} in the expansion of $(a + b)^{13}$
   (d) the coefficient of a^2b^{10} in the expansion of $(a + 2b)^{12}$

18. (a) Prove combinatorially that if n is odd, then the number of ways to select an even number of objects from n is equal to the number of ways to select an odd number of objects.
   (b) Give a combinatorial proof of Vandermonde's identity: for positive integers m and n, and r an integer such that $0 \le r \le n + m$,

$$\binom{n+m}{r} = \binom{n}{0}\binom{m}{r}+\binom{n}{1}\binom{m}{r-1}+\binom{n}{2}\binom{m}{r-2}+\cdots+\binom{n}{r}\binom{m}{0}.$$

   (c) Prove that $\binom{2n}{n}+\binom{2n}{n+1}=\frac{1}{2}\binom{2n+2}{n+1}$.