Solutions to Assignment #7

4.5 #6

$\left\{\begin{pmatrix}3\\6\\-9\\-3\end{pmatrix}, \begin{pmatrix}6\\-2\\5\\1\end{pmatrix}, \begin{pmatrix}-1\\-2\\3\\1\end{pmatrix}\right\}$ spans the space, but

$$\begin{pmatrix}3 & 6 & -1\\6 & -2 & -2\\-9 & 5 & 3\\-3 & 1 & 1\end{pmatrix} \sim \begin{pmatrix}3 & 6 & -1\\0 & -14 & 0\\0 & 23 & 0\\0 & 7 & 0\end{pmatrix} \sim \begin{pmatrix}3 & 0 & -1\\0 & 1 & 0\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$$

So $-\frac{1}{3}\begin{pmatrix}3\\6\\-9\\-3\end{pmatrix} = \begin{pmatrix}-1\\-2\\3\\1\end{pmatrix}$ so a basis for the spance is $\left\{\begin{pmatrix}6\\-2\\5\\1\end{pmatrix}, \begin{pmatrix}-1\\-2\\3\\1\end{pmatrix}\right\}$ so the dimension is 2.

4.5 #14

$$\dim \text{Nul } A = 3$$
$$\dim \text{Col } A = 3$$

4.5 #24

$$[p]_{\mathcal{B}} = \begin{pmatrix}1 & 1 & 2\\0 & -1 & -4\\0 & 0 & 1\end{pmatrix}^{-1} \begin{pmatrix}7\\-8\\3\end{pmatrix}$$

so

$$\begin{pmatrix}1 & 1 & 2 & 7\\0 & -1 & -4 & -8\\0 & 0 & 1 & 3\end{pmatrix} \sim \begin{pmatrix}1 & 1 & 0 & 1\\0 & -1 & 0 & 4\\0 & 0 & 1 & 3\end{pmatrix} \sim \begin{pmatrix}1 & 0 & 0 & 5\\0 & 1 & 0 & -4\\0 & 0 & 1 & 3\end{pmatrix}$$

So $[p]_{\mathcal{B}} = \begin{pmatrix}5\\-4\\3\end{pmatrix}$.

4.6 #2

rank $A = 3$

dim Nul $A = 2$

A basis for Col A is $\left\{\begin{pmatrix}1\\-2\\-3\\3\end{pmatrix}, \begin{pmatrix}4\\-6\\-6\\4\end{pmatrix}, \begin{pmatrix}9\\-10\\-3\\0\end{pmatrix}\right\}$

A basis for Row A is $\{(1\ \ -3\ 0\ 5\ \ -7), (0\ 0\ 2\ \ -3\ 8), (0\ 0\ 0\ 0\ 5)\}$

A basis for Nul A is $\left\{\begin{pmatrix}3\\1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}-5\\0\\3\\1\\0\end{pmatrix}\right\}$

4.6 #8

dim Nul $A = 2$, but Col $A \leq \mathbb{R}^6$ so Col $A \neq \mathbb{R}^4$.

4.6 #16

$$\dim \text{Nul } A \geq 0$$

4.6 #30

In order fo $Ax = b$ to be consistent, $[A\ b]$ must not have a pivot in the last column. Therefore the number of pivots of A is the same as the number of pivots of $[A\ b]$. Therefore Rank $A =$ Rank $[A\ b]$.

4.7 #6

a.

$$\underset{\mathcal{D}\leftarrow\mathcal{F}}{\mathcal{P}} = \begin{pmatrix} 2 & 0 & -3 \\ -1 & 3 & 0 \\ 1 & 1 & 2 \end{pmatrix}$$

b.

$$[x]_{\mathcal{D}} = \underset{\mathcal{D}\leftarrow\mathcal{F}}{\mathcal{P}}[x]_{\mathcal{F}} = \begin{pmatrix} 2 & 0 & -3 \\ -1 & 3 & 0 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ -7 \\ 3 \end{pmatrix}$$

4.7 #8

$$\begin{pmatrix} -1 & 1 & 1 & 1 \\ 8 & -5 & 4 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -1 & -1 \\ 0 & 3 & 12 & 9 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -1 & -1 \\ 0 & 1 & 4 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 4 & 3 \end{pmatrix}$$

So

$$\underset{\mathcal{B}\leftarrow\mathcal{C}}{\mathcal{P}} = \begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}$$

and

$$\underset{\mathcal{C}\leftarrow\mathcal{B}}{\mathcal{P}} = \underset{\mathcal{B}\leftarrow\mathcal{C}}{\mathcal{P}}^{-1} = \begin{pmatrix} 3 & -2 \\ -4 & 3 \end{pmatrix}$$