Solutions to Assignment #6

4.3 #4

$$\begin{pmatrix} 2 & 1 & -7 \\ -2 & -3 & 5 \\ 1 & 2 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 4 \\ -2 & -3 & 5 \\ 2 & 1 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 13 \\ 0 & -3 & -15 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 13 \\ 0 & 0 & -24 \end{pmatrix}$$

The matrix has three pivots, so $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}, \begin{pmatrix} -7 \\ 5 \\ 4 \end{pmatrix}$ are linearly independent, and they span $\mathbb{R}^3$, so they form a basis for $\mathbb{R}^3$.

4.3 #14

Vectors in Nul A look like

$$\begin{pmatrix} -2x_2 - 4x_4 \\ x_2 \\ \frac{7}{5}x_4 \\ x_4 \\ 0 \end{pmatrix} = x_2 \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -4 \\ 0 \\ \frac{7}{5} \\ 1 \\ 0 \end{pmatrix}$$

So $\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -4 \\ 0 \\ \frac{7}{5} \\ 1 \\ 0 \end{pmatrix} \right\}$ is a basis for Nul A.

The pivot columns of A form a basis for Col A, so $\left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} -5 \\ -5 \\ 0 \\ -5 \end{pmatrix}, \begin{pmatrix} -3 \\ 2 \\ 5 \\ -2 \end{pmatrix} \right\}$ form a basis for Col A.

4.3 #24

Consider the $n \times n$ matrix $V = [v_1 \ \dots \ v_n]$ Then since $\{v_1, \dots, v_n\}$ are linearly independent, the V is invertible, by the IMT. Also by the IMT, the columns fo V namely $\{v_1, \dots, v_n\}$ span $\mathbb{R}^n$. Therefore $\{v_1, \dots, v_n\}$ is a basis for $\mathbb{R}^n$.

4.4 #4

$$x = -4 \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} + 8 \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix} - 7 \begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 + 24 - 28 \\ -8 - 40 + 49 \\ 0 + 16 - 21 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -5 \end{bmatrix}$$

4.4 #6

$$\begin{pmatrix} 1 & 5 & 4 \\ -2 & -6 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 5 & 4 \\ 0 & 4 & 8 \end{pmatrix} \sim \begin{pmatrix} 1 & 5 & 4 \\ 0 & 1 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -6 \\ 0 & 1 & 2 \end{pmatrix}$$

So $[x]_{\mathcal{B}} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$.

4.4 #10

$$\mathcal{P}_{\mathcal{B}} = \begin{pmatrix} 3 & 2 & 8 \\ -1 & 0 & -2 \\ 4 & -5 & 7 \end{pmatrix}$$

4.4 #32 Let $\mathcal{E} = \{1, t, t^2\}$ be the standard basis for $\mathbb{P}_2$. Notice that $[p_1]_{\mathcal{E}} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $[p_2]_{\mathcal{E}} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ and $[p_3]_{\mathcal{E}} = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}$. Then

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 1 & 3 & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 0 & 1 & -5 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

This has 3 pivots so the columns form a basis for $\mathbb{R}^3$, therefore $\{p_1, p_2, p_3\}$ form a basis for $\mathbb{P}_2$.

$$[q]_{\mathcal{E}} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 1 & 3 & -4 \end{pmatrix} \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -8 \end{pmatrix}$$

So $q = 1 + 3t - 8t^2$.