Solutions to Assignment #5

4.1 #6

Notice that the 0 polynomial is not of the form $a + t^2$, for a in $\mathbb{R}$. Therefore polynomials of this form do not form a subspace of $\mathbb{P}_n$.

4.1 #12

$$\begin{bmatrix} s+3t \\ s-t \\ 2s-t \\ 4t \end{bmatrix} = s\begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix} + t\begin{bmatrix} 3 \\ -1 \\ -1 \\ 4 \end{bmatrix}$$

Therefore vectors of this form are all the vectors in Span $\left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \\ -1 \\ 4 \end{pmatrix} \right\}$. Since the span of a set of vectors is always a subspance, then vectors of this form must be a subspace as well.

4.1 #18

Vectors of this form are the vectors of Span $\left\{ \begin{pmatrix} 4 \\ 0 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} \right\}$

4.2 #6

$$\begin{bmatrix} 1 & 5 & -4 & -3 & 1 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 6 & -8 & 1 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So

$$x_1 + 6x_3 - 8x_4 + x_5 = 0$$
$$x_2 - 2x_3 + x_4 = 0$$
$$x_1 = -6x_3 + 8x_4 - x_5$$
$$x_2 = 2x_3 - x_4$$

Vectors in Nul(A) have the form

$$\begin{bmatrix} -6x_3 + 8x_4 - x_5 \\ 2x_3 - x_4 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_3\begin{bmatrix} -6 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} 8 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_5\begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

So Nul$(A) = $ Span $\left\{ \begin{pmatrix} -6 \\ 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 8 \\ -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$.

4.2 #22

Certainly $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is in Col(A), since it is the first column of A.

$$\begin{pmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & 4 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -7 & 6 \\ 0 & 1 & 4 & -2 \end{pmatrix}$$

$$x_1 = 7x_3 - 6x_4$$
$$x_2 = -4x_3 + 2x_4$$

A vector in Nul(A) looks like $x_3 \begin{pmatrix} 7 \\ -4 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -6 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ so $\begin{pmatrix} 7 \\ -4 \\ 1 \\ 0 \end{pmatrix}$ is in Nul(A).

4.2 #24

$$\begin{pmatrix} -8 & -2 & -9 & 2 \\ 6 & 4 & 8 & 1 \\ 4 & 0 & 4 & -2 \end{pmatrix} \sim \begin{pmatrix} 2 & 0 & 2 & -1 \\ -8 & -2 & -9 & 2 \\ 6 & 4 & 8 & 1 \end{pmatrix} \sim \begin{pmatrix} 2 & 0 & 2 & -1 \\ 0 & -2 & -1 & -2 \\ 0 & 4 & 2 & 4 \end{pmatrix} \sim \begin{pmatrix} 2 & 0 & 2 & -1 \\ 0 & -2 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

So the system is consistent, thus w is in Col(A).

$$Aw = \begin{pmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \bar{0}$$

So w is in Nul(A) as well.