Answers to Even Assigned Problems in Homework Assignment #4

Section 2.3

Problem #	Answer
2	Not invertible, by Theorem 4 in section 2.2, because the determinant is zero. Less obvious is the fact that the columns are linearly dependent - the second column is $-\frac{3}{2}$ times the first column. From this and the IMT, it follows that the matrix is singular.
4	The matrix obviously has linearly dependent columns (because one column is zero), and so the matrix is not invertibe, by (e) of the IMT.
6	Not invertible, by the IMT. The matrix row reduces to $\begin{pmatrix} \mathbf{1} & -5 & -4 \\ 0 & \mathbf{3} & 4 \\ 0 & 0 & 0 \end{pmatrix}$ and is not row equivalent to I_3.
16	No, because statement (h) of the IMT is then false. A 5×5 matrix cannot be invertible when its columns do not span $\mathbb{R}^5$.
20	By the boz following the IMT, E and F are invertible and are inverses. So $FE = I = EF >$ Thus E and F commute.
22	Statement (g) of the IMT is false for H, so statemtent (d) is false, too. That is, the equation $Hx = 0$ has a notrivial solution
24	The equation $Lx = 0$ *always* has the trivial solution. This fact gives no information about the columns of L.

Section 2.4

Problem #	Answer
2	$\begin{pmatrix} EA & EB \\ FC & FD \end{pmatrix}$
4	$\begin{pmatrix} A & B \\ -XA + C & -XB + D \end{pmatrix}$
6	$X = A^{-1}$ (by the IMT, because A is square), $Z = C^{-1}$(by the IMT, because C is square), $y = -C^{-1}BA^{-1}$
8	$X = A^{-1}$ (by the IMT, because A is square), $Y = 0$, $Z = -A^{-1}B$
10	$X = -A + BC, Y = -B, Z = -C$

Section 2.5

Problem #	Answer
10	$\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & -5 & 1 \end{pmatrix} \begin{pmatrix} -5 & 3 & 4 \\ 0 & -2 & -1 \\ 0 & 0 & 9 \end{pmatrix}$
14	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ -2 & -1 & 1 & 0 \\ -1 & -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 4 & -1 & 5 \\ 0 & -5 & 1 & -6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
16	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 & 0 \\ \frac{3}{2} & -2 & 1 & 0 & 0 \\ -3 & -2 & 0 & 1 & 0 \\ 4 & -3 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -6 & 6 \\ 0 & -7 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
22	$B = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & 2 & -1 \\ -3 & -3 & 2 \end{pmatrix}, C = \begin{pmatrix} 2 & -4 & -2 & 3 \\ 0 & 3 & 1 & -1 \\ 0 & 0 & 0 & 5 \end{pmatrix}$; if $A = LU$, with only three nonzero rows in U, use the first three columns of L for B and the top three rows of U for C.
24	Since Q is square that $Q^TQ = I$, Q is invertibel and $Q^{-1} = Q^T$, by the Invertible matrix Theorem. Thus A is the product of invertible matrices and hence is invertible. By Theorem 5, the equation $Ax = b$ has a unique solution for all b. Form $Ax = b$, we have $QRx = b, Q^TQRx = Q^Tb$, $Rx = Q^Tb$, and $x = R^{-1}Q^Tb$. A good algorithm for finding b is to compute Q^Tb and then row reduce $[R\ Q^Tb]$. (See Exercise 12 in Section 2.2.) The reduction is fast because R is triangular.

Section 3.1

Problem #	Answer
12	36. Start with row 1 or column 4.
14	9. Start with row 4 or column 5.
20	$ad - bc, a(kd) - b(kc) = k(ad - bc)$. Scaling a row by k multiplies the determinant by k.
22	$ad - bc$, $(ad + kcd) - (bc + kdc) = ad - bc$. Row replacement does not change a determinant.
26	1
28	k
30	-1
32	k. A scaling matrix is diagonal, with k on the diagonal and with 1's as the other diagonal entries. The determinant is the product of the diagonal entries.
34	$\det EA = \begin{vmatrix} a & b \\ kc & kd \end{vmatrix} = akd - bkc = k(ad - bc)$ $= (\det E)(\det A)$
36	$\det EA = \begin{vmatrix} a & b \\ ka + c & kb + d \end{vmatrix} = a(kb - d) - b(ka + c)$ $= akb + ad - bka - bc = (+1)(ad - bc)$ $= (\det E)(\det A)$

Section 3.2

Problem #	Answer
6	-18
8	0
10	24
14	0
16	21
20	7
22	Not invertible
32	$\det(rA) = r^n \cdot \det(A)$
36	$0 = \det(A^4) = (\det(A))^4$, by Theorem 6. So $\det A = 0$, which implies that A is not invertible by Theorem 4.