Answers to Even Assigned Problems in Homework Assignment #3

Section 1.9

Problem #	Answer
18	$\begin{pmatrix} -3 & 2 \\ 1 & -4 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$
26	The transformation in Exercise 2 is not one-to-one, by Theorem 12, because the standard matrix is 2×3 and so has linearly dependent columns. However, the matrix has a pivot in each row and so the columns span $\mathbb{R}^2$. By Theorem 12, the transformation maps $\mathbb{R}^3$ onto $\mathbb{R}^2$.
28	The standard matrix A fo the transformation T in Exercise 14 has linearly independent columns, because Figure 6 shows that a_1 and a_2 are not multiples. So T is one-to-one, by Theorem 12. Also, A must have a pivot in each column because the equation $Ax = 0$ has no free variables. Thus, the echelon form of A is $\begin{pmatrix} \square & * \\ 0 & \square \end{pmatrix}$. Since A has a pivot in each row, the columns of A span $\mathbb{R}^2$. So T maps $\mathbb{R}^2$ onto $\mathbb{R}^2$. An alternative arguement for the second part is to observe directly from Fig. 6 that a_1 and a_2 spahn $\mathbb{R}^2$. This is more or less evident, based on experience with grids such as those in Fig. 8 and the fugure with Exercises 7 and 8 in Section 1.3.
32	A has m pivot columns if and only if A has a pivot position in each row. By Theorem 4 in Section 1.4, this happens if and only if the columns of A span $\mathbb{R}^m$, and this in turn happens (by Thoerem 12) if and only if T maps $\mathbb{R}^n$ onto $\mathbb{R}^m$.

Section 2.1

Problem #	Answer
4	$A - 5I_3 = \begin{pmatrix} 4 & -1 & 3 \\ -8 & 2 & -6 \\ -4 & 1 & 3 \end{pmatrix}$ $(5I_3)A = \begin{pmatrix} 45 & -5 & 15 \\ -40 & 35 & -30 \\ -20 & 5 & 40 \end{pmatrix}$
18	The first two columns of AB are Ab_1 and Ab_2. They are equal because b_1 and b_2 are equal.
22	If the columns of B are linearly dependent, then there exists a nonzero vector x such that $Bx = 0$. Form this, $A(Bx) = A0$ and $(AB)x = 0$ (by associativity). Since x is nonzero, the columns of AB must be linearly dependent.
24	Take any b in $\mathbb{R}^m$. By hypothesis, $ADb = I_m b = b$. Rewrite this equation as $A(Db) = b$. Thus, the vector $x = Db$ satisfies $A = b$. This proves that the equation $Ax = b$ has a solution for each b in $\mathbb{R}^m$. By Theorem 4 in section 1.4, A has a pivot position in each row. Since each pivot is in a different column, A must have at least as many columns as rows.
26	Write $I_3 = [e_1\ e_2\ e_3]$ and $D = [d_1\ d_2\ d_3]$. by definition of AD, the equation $AD = I_3$ is equivalent to the three equations $Ad_1 = e_1, Ad_2 = e_2$, and $Ad_3 = e_3$. Each of these equations has at least one solution because the columns fo A span $\mathbb{R}^3$. (See Theorem 4 in Section 1.4). Select one solution of each equation, and use them for the columns of D. Then $AD = I_3$.

Section 2.2

Problem #	Answer
2	$\begin{pmatrix} -2 & 1 \\ \frac{7}{2} & -\frac{3}{2} \end{pmatrix}$
4	$\frac{1}{4}\begin{pmatrix} -8 & 4 \\ -7 & 3 \end{pmatrix}$ or $\begin{pmatrix} -2 & 1 \\ -\frac{7}{4} & \frac{3}{4} \end{pmatrix}$
14	Right multiply each side of the equation $(B-C)D = 0$ by D^{-1}, and obtain $(B-C)DD^{-1} = 0D^{-1}, (B-C)I = 0$ Thus $B - C = 0$, and $B = C$
16	Let $C = AB$. Since B is invertible, use B^{-1} to solve for A: $CB^{-1} = ABB^{-1}, CB^{-1} = AI = A$ This shows that A is the product of invertible matrices and hence is invertible, by Theorem 6.
18	Left-multiply each side of $A = PBP^{-1}$: $P^{-1}A = P^{-1}PBP^{-1}, P^{-1}A = IBP^{-1}, P^{-1}A = BP^{-1}$ Then right-multiply each side of the result by P: $P^{-1}AP = BP^{-1}P, P^{-1}AP = BI, P^{-1}AP = B$.
20	a. Left-multiply both isdes of $(A - AX)^{-1} = X^{-1}B$ by X to see that B is invertible because it is the product of invertible matrices. b. $X = (A + B^{-1})^{-1}A$. A careful proof should justify $A - AX = B^{-1}X$, and show that $A + B^{-1}$ is invertible.
24	If the equation $Ax = b$ has a solution for each b in $\mathbb{R}^n$, then A has a pivot position in each row, by Theorem 4 in Section 1.4. Since A is square, the pivots must be on the diagonal of A. It follows that A is row equivalent to I_n. By Theorem 7, A is invertible.