

## Answers to Even Assigned Problems in Homework Assignment #2

### Section 1.3

Problem #	Answer
12	No, $b$ is <i>not</i> a linear combination of $a_1, a_2$ , and $a_3$ .
18	$h = -\frac{7}{2}$

### Section 1.4

Problem #	Answer
14	No. The equation $Ax = u$ has no solution.
18	ONLY three rows contain a pivot position. The equation $Bx = y$ does <i>not</i> have a solution for each $y$ in $\mathbb{R}^4$ , by Theorem 4.
28	The matrix can be written as $c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 + c_5v_5 = v_6$ , where $c_1 = -3$ $c_2 = 2, c_3 = 4, c_4 = -1, c_5 = 2$ , and $v_1 = \begin{bmatrix} -3 \\ 5 \\ 9 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} 5 \\ 8 \\ 7 \\ -4 \end{bmatrix}, v_3 = \begin{bmatrix} -4 \\ 1 \\ 8 \\ -1 \end{bmatrix},$ $v_4 = \begin{bmatrix} 5 \\ 8 \\ 7 \\ -4 \end{bmatrix}, v_5 = \begin{bmatrix} -4 \\ 1 \\ 8 \\ -1 \end{bmatrix}, v_6 = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$
32	A set of three vectors in $\mathbb{R}^4$ cannot span $\mathbb{R}^4$ . Reason: The matrix $A$ whose columns are these three vectors has four rows. To have a pivot in each row, $A$ would have to have at least four columns (one for each pivot), which is not the case. Since $A$ does not have a pivot in every row, its columns do not span $\mathbb{R}^4$ , by Theorem 4. In general, a set of $n$ vectors in $\mathbb{R}^m$ cannot span $\mathbb{R}^m$ when $n$ is less than $m$ .

### Section 1.5

Problem #	Answer
4	The variable $x_3$ is free; the system has nontrivial solutions.

**Section 1.7**

Problem #	Answer
10	a. No $h$ b. All $h$
16	Lin. depen.
18	Lin. depen.
20	Lin. depen.
24	$\begin{pmatrix} \square & * \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & \square \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$
28	It must have 5 pivot columns. (By Theorem 4 in Section 1.4, $A$ has a pivot in each of its five rows. Since each pivot position is in a different column, $A$ has five pivot columns.)
30	a. $n$ b. The columns of $A$ are linearly independent if and only if the equation $Ax = 0$ has only the trivial solution. This happens if and only if $Ax = 0$ has no free variables, which in turn happens if and only if every variable is a basic variable, that is, if and only if every column of $A$ is a pivot column.