Math 3000 Intro to Abstract Math
Quiz # 8, Spring 2006

Name: **Solutions**

1. Let $R = \{(1,5),(2,2),(3,4),(5,2)\}$, $S = \{(2,4),(3,4),(3,1),(5,5)\}$, and $T = \{(1,4),(3,5),(4,1)\}$. Find

   b. $R \circ T$.
   $$R \circ T = \{(3,2),(4,5)\}$$

   d. $R \circ R$.
   $$R \circ R = \{(1,2),(5,2)\}$$

   f. $T \circ T$.
   $$T \circ T = \{(1,1),(4,4)\}$$

   h. $(R \circ S) \circ T$.
   $$(R \circ S) \circ T = \{(3,2)\}$$

2. Suppose that R and S are equivalence relations on a set A. Prove that $R \cap S$ is an equivalence relation on A.

   Proof: (Reflexive:) Let $a \in A$, then since R is reflexive, then $(a,a) \in R$. Also, since S is reflexive, then $(a,a) \in R$. Therefore $(a,a) \in R \cap S$ so $R \cap S$ is reflexive.
   (Symmetric:) Now suppose that $(a,b) \in S \cap R$, then $(a,b) \in S$, and $(a,b) \in R$, but since R and S are both symmetric, then $(b,a) \in R$, and $(b,a) \in S$. So $(b,a) \in R \cap S$, and $R \cap S$ is symmetric.
   (Transitive:) Now suppose that $(a,b) \in R \cap S$, and $(b,c) \in R \cap S$, then since $(a,b) \in R$, and $(b,c) \in R$, and R is transitive, then $(a,c) \in R$. Similarly, since $(a,b) \in S$, and $(b,c) \in S$ and S is transitive, then $(a,c) \in S$. Therefore $(a,c) \in R \cap S$. So $R \cap S$ is transitive. Thus $R \cap S$ is an equivalence relation on A. ■