Math 3000 Intro to Abstract Math
Quiz # 6, Spring 2006

Name: **Solutions**

1. Let A, B, and C be sets. Prove that if $A \subseteq C$ and $B \subseteq C$, then $A \cup B \subseteq C$.

   Proof: Assume that $A \subseteq C$ and $B \subseteq C$, then let $x \in A \cup B$. Then we have that $x \in A$ or $x \in B$. If $x \in A$, then since $A \subseteq C$, then $x \in C$. Similarly, if $x \in B$, then since $B \subseteq C$, then $x \in C$. So in both cases $x \in C$. Therefore we have shown that $A \cup B \subseteq C$. ∎

2. Let A, B, C, and D be sets with $C \subseteq A$, and $D \subseteq B$. Prove that if A and B are disjoint, then C and D are disjoint.

   Proof: Assume that A, B, C, and D are sets with $C \subseteq A$, and $D \subseteq B$. Also, suppose that A and B are disjoint. This means that $A \cap B\ \emptyset$. As an intermediate step, we now show that $C \cap D \subseteq A \cap B$. To do this we let $x \in C \cap D$. Then $x \in C$, and $x \in D$. However, since $C \subseteq A$, and $D \subseteq B$, then $x \in A$, and $x \in B$ as well. So $C \cap D \subseteq A \cap B$, but $A \cap B = \emptyset$. Therefore $C \cap D \subseteq \emptyset$. This of course is only true when $C \cap D = \emptyset$. Therefore we have shown that C and D are disjoint. ∎