Math 3000 Intro to Abstract Math
Quiz # 5, Spring 2006

Name: **Solutions**

1. Provide either a proof or a counterexample for the following statement:
   For all positive integers x, $x^2 + x + 41$ is a prime.
   This statement is false.

   Proof: Let $x = 41$. Then notice that x is a positive integer, and
   $$x^2 + x + 41 = 41^2 + 41 + 41 = 41(41 + 1 + 1) = 41(43)$$
   which clearly is not prime. Therefore we have produced a counterexample. ■

2. Let a and b be natural numbers, and let $\text{LCM}(a, b) = m$. Prove that $\text{LCM}(a, b) = b$ if and only if a divides b.

   Proof: ($\Rightarrow$) Assume that $\text{LCM}(a, b) = b$. Then we get directly from the definition that $a \mid b$.
   ($\Leftarrow$) So now suppose that $a \mid b$. Then since $b \mid b$, then b is a common multiple of a and b. Now let n be any other common multiple of a and b, then by definition $a \mid n$, and $b \mid n$, therefore b is the least common multiple of a and b.
   So we have shown that $\text{LCM}(a, b) = b$ if and only if a divides b. ■