

1. Let x be an integer. Write a proof by contraposition to show that if x^2 is not divisible by 4, then x is odd.

Proof: (By Contrapositive) Assume that x is even, then $x = 2k$ for some integer k . This give us that

$$x^2 = (2k)^2 = 4k^2$$

where k^2 is an integer. Therefore, $4 \mid x^2$. So by contrapositive, if x^2 is not divisible by 4, then x is odd.

■

2. Suppose a , b , and $c \neq 0$ are integers. Write a proof of the following biconditional statement:
 ac divides bc if and only if a divides b .

Proof: (\Rightarrow) Assume that $ac \mid bc$, then $bc = ack$ for some integer k . Dividing both sides by $c \neq 0$, gives us $b = ak$ which implies that $a \mid b$ as desired.

(\Leftarrow) Now suppose that $a \mid b$, then $b = ak$ for some integer k . This gives us that $bc = ack$, and thus that $ac \mid bc$. ■