Math 3000 Intro to Abstract Math
Quiz # 3, Spring 2006

Name: **Solutions**

1. Prove by cases that if n is a natural number, $n^2 + n + 3$ is odd.

   Proof: Assume that n is a natural number. Then we have that n is either even or odd.
   (Case 1: n is even) Suppose that n is even, then $n = 2k$ for some integer k. In this case

$$\begin{aligned} n^2 + n + 3 + 3 &= (2k)^2 + (2k) + 2 + 1 \\ &= 2(2k^2 + k + 1) + 1 \end{aligned}$$

   where $2k^2 + k + 1$ is an integer. Therefore $n^2 + n + 3$ is odd in this case.
   (Case 2: n is odd) Now suppose that n is odd, then $n = 2k + 1$ for some integer k. In this case

$$\begin{aligned} n^2 + n + 3 + 3 &= (2k+1)^2 + (2k+1) + 3 \\ &= 4k^2 + 4k + 1 + 2k + 1 + 3 \\ &= 4k^2 + 6k + 5 \\ &= 2(2k^2 + 3k + 2) + 1 \end{aligned}$$

   where $2k^2 + 3k + 2$ is an ingeger. Therefore $n^2 + n + 3$ is odd in this case as well. ■

2. Use the technique of working backward form the desired conclusion to prove that if $x^3 + 2x^2 < 0$, then $2x + 5 < 11$.
   (Scratch Work)

$$\begin{aligned} 2x + 5 &< 11 \\ 2x &< 6 \\ x &< 3 \\ x^3 + 2x^2 &< 0 \\ x^2(x+2) &< 0 \\ x &< -2 \end{aligned}$$

   Proof: Assume that $x^3 + 2x^2 < 0$. Then notice that

$$\begin{aligned} x^3 + 2x^2 &< 0 \quad \Rightarrow \\ x^2(x+2) &< 0, \end{aligned}$$

   but since $x^2 \geq 0$, then we must have $x < -2$. Now $x < -2$ means $x < 3$ as well, and this gives us

$$\begin{aligned} x &< 3 \quad \Rightarrow \\ 2x &< 6 \quad \Rightarrow \\ 2x + 5 &< 11 \end{aligned}$$

   as desired. ■