

Math 2411 – Calculus II Technology Final Exam

This is a take-home exam and you may use books, notes, or computing devices. Please work carefully and write out your solutions in a neat and organized way. Begin each solution on a separate sheet of paper. Your instructor will give you the due date for the exam. Work **three** of the following **four** problems and indicate clearly which problems you wish to have graded. **Please talk to no one else about the exam and do not ask anyone else for help.**

1. Consider the functions $f(x) = x^3 - 2x^2 + 2$ and $g(x) = -x^3 + 4x + 2$.
 - a. Graph f and g on the same set of axes.
 - b. Find the area of the *entire region* bounded by the two curves.
 - c. Suppose the region between these curves *in the first quadrant* is revolved about the x -axis. What is the volume of the solid that is produced?
 - d. Approximate the length of the curve $y = f(x)$ between $x = 0$ and $x = 2$.
2. One of many ways to approximate the constant $\pi/4 \approx 0.7853981633$ is with the infinite series

$$\frac{\pi}{4} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2k-1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

- a. Make a table similar to the one below showing the first 10 partial sums of the series. Each entry should be correct to three decimal places.

n	S_n	n	S_n
1		6	
2		7	
3		8	
4		9	
5		10	

- b. Compute the 50th partial sum, S_{50} , and use it to approximate $\pi/4$. What is the error in this approximation, $|\pi/4 - S_{50}|$?
 - c. How many terms of the series must be used to approximate $\pi/4$ with an error less than 10^{-4} ?
3. Consider the function $f(x) = \sqrt{1+x} = (1+x)^{1/2}$.
 - a. Graph the function f on the interval $-1 < x < 1$.
 - b. Find the Taylor polynomials $p_0, p_1, p_2,$ and p_3 of degree 0, 1, 2, 3 for f about $x = 0$.
 - c. On the same axes as in part (a), graph these four polynomials.

d. Compute the approximations to $\sqrt{1.05}$ given by the four Taylor polynomials found above (four approximations).

4. Two water tanks are shaped like right circular cones. Tank A has a radius of 4 meters and a height of 10 meters. Tank B has a radius of 2 meters and a height of 40 meters. You can verify that both tanks hold the same amount of water when they are full. However, which tank requires more work to empty its contents by pumping the water over the top of the tank? Compute the amount of work required to pump the water out of each tank (in joules) and compare.

