Assignment # 1

Due 6-5-06

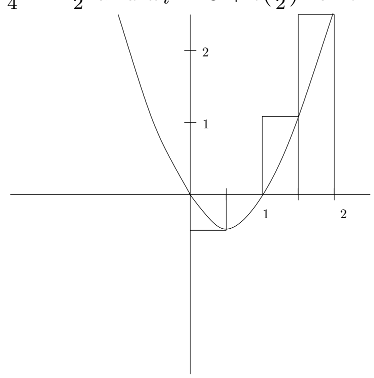
(1) (a) Evaluate the Riemann sum for

$$f(x) = x^2 - x \quad 0 \leq x \leq 2$$

with four subintervals, taking the sample points to be right endpoints.

Explain, with the aid of a diagram, what the Riemann sum represents.

Solution: $\Delta x = \frac{2-0}{4} = \frac{1}{2}$ and $x_i = 0 + i(\frac{1}{2})$ for $i = 1, 2, 3, 4$.



$$\begin{aligned} R_4 &= \sum_{i=1}^{4} (\frac{1}{2}) f(i(\frac{1}{2})) \\ &= \sum_{i=1}^{4} (\frac{1}{2}) \left[(\frac{i}{2})^2 - \frac{i}{2} \right] \\ &= \frac{5}{4} \end{aligned}$$

(b) Use the definition of a definite integral to calculate the value of the integral

$$\int_0^2 (x^2 - x)\ dx$$

To do this, write the definite integral as the limit of the Riemann sum, and use a Computer Algebra System to evaluate it.

Solution:

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} (\frac{2}{n}) \left[(\frac{2i}{n})^2 - (\frac{2i}{n}) \right] = \frac{2}{3}$$

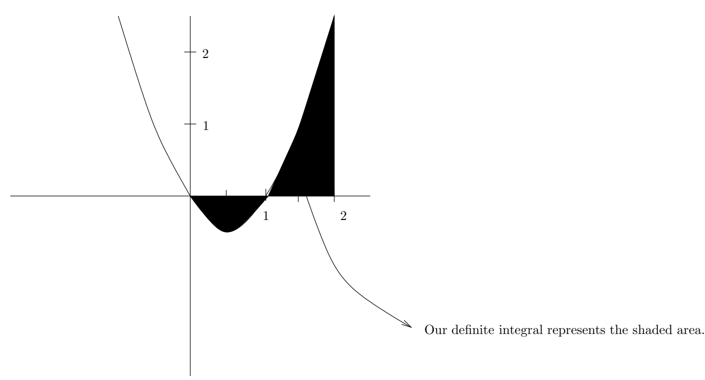
(c) Use the Fundamental Theorem of Calculus to check your answer in part b).

Solution:

$$\int_0^2 (x^2 - x)dx = \frac{x^3}{3} - \frac{x^2}{2}\Big|_0^2$$
$$= (\frac{8}{3} - \frac{4}{2}) = \frac{16-12}{6} = \frac{4}{6} = \frac{2}{3}$$

(d) Draw a diagram to explain the geometric meaning of the integral in part b).

Solution:



(2) Evaluate

(a) $\int_0^1 \frac{d}{dx}(e^{\arctan x})\ dx$

Solution:

$$\int_0^1 \frac{d}{dx} e^{\arctan x} dx = e^{\arctan x}\big|_0^1$$
$$= e^{\frac{\pi}{4}} - e^0 = e^{\frac{\pi}{4}} - 1$$

(b) $\dfrac{d}{dx}\displaystyle\int_0^1 e^{\arctan x}\ dx$

Solution:

$$\frac{d}{dx}\int_0^1 e^{\arctan x} dx = 0$$

since $\int_0^1 e^{\arctan x} dx$ is a constant.

(c) $\dfrac{d}{dx}\displaystyle\int_0^x e^{\arctan t}\ dt$

Solution:

$$\frac{d}{dx}\int_0^x e^{\arctan t} dt = e^{\arctan x}$$

by part 1 of the Fundamental Theorem of Calculus.

Please note that although these look similar, they are all quite different.

(3) Evaluate the integral

$$\int_0^T (x^4 - 8x + 7)\ dx$$

Solution:

$$\begin{aligned}\int_0^T (x^4 - 8x + 7dx &= \frac{x^5}{5} - \frac{8x^2}{2} + 7x\Big|_0^T \\ &= \frac{1}{5}T^5 - 4T^2 + 7T\end{aligned}$$

(4) Evaluate the integral

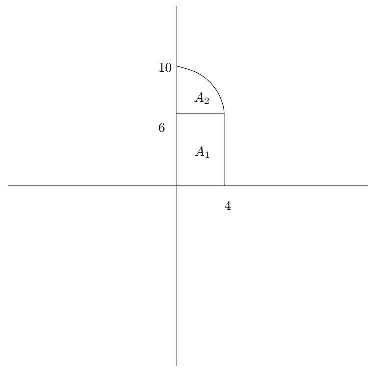
$$\int_1^4 \frac{x^2 - x + 1}{\sqrt{x}}\ dx$$

Solution:

$$\begin{aligned}\int_1^4 \frac{x^2 - x + 1}{\sqrt{x}} dx &= \int_1^4 x^{\frac{3}{2}} - x^{\frac{1}{2}} + x^{-\frac{1}{2}} dx \\ &= \frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}}\Big|_1^4 \\ &= \left[\frac{2}{5}(32) - \frac{2}{3}(8) + 2(2)\right] - \left[\frac{2}{5} - \frac{2}{3} + 2\right] \\ &= \frac{146}{15}\end{aligned}$$

(5) If $f(x) = 6 + \sqrt{(16 - x^2)}$, sketch a graph of f, and use properties of definite integrals and geometry to evaluate the area under the curve $f(x)$ from 0 to 4.

Solution: $f(x) = 6 + \sqrt{16 - x^2}$.



$$
\begin{aligned}
\int_0^4 f(x)dx &= A_1 + A_2 \\
&= 4 \cdot 6 + \frac{1}{4} \cdot \pi \cdot 4^2 \\
&= 24 + 4\pi
\end{aligned}
$$

(6) Calculate y' for

$$y = \ln(x^2 e^x)$$

Solution:

$$
\begin{aligned}
y &= \ln(x^2 e^x) \\
y' &= \frac{1}{x^2 e^x}(2xe^x + x^2 e^x) \\
&= \frac{xe^x(2+x)}{x^2 e^x} = \frac{x+2}{x}
\end{aligned}
$$

(7) Calculate y' for

$$y = \arctan(\arcsin\sqrt{x})$$

Solution: There are several possible solutions to this problem.

$$\begin{aligned} y &= \arctan(\arcsin\sqrt{x}) \\ y' &= \frac{1}{1+(\arcsin\sqrt{x})^2} \cdot \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} \\ &= \frac{1}{2(1+(\arcsin\sqrt{x})^2)(\sqrt{x-x^2})} \end{aligned}$$