Math 2411 Calculus II
Practice Final, Summer 2006

Name________________________________________

1. The first five terms of a sequence are written below.

$$\frac{1}{3}, \frac{4}{4}, \frac{9}{5}, \frac{16}{6}, \frac{25}{7}, \frac{36}{8}, \ldots$$

   (a) Write an expression which describes the rule for the $n$th term. $a_n =$ ________________________

   (b) Determine if the sequence converges or diverges. If the sequence converges, find the value it converges to. Show your work.

2. Consider the series defined as $\displaystyle\sum_{n=1}^{\infty} \frac{2}{(4n^2-1)^2}$

   (a) Do a **partial fraction decomposition** to rewrite the rule for $a_n$ of the series.

   (b) Write the first **4 partial sums** of the series, then determine a rule for the $n$**th partial sum.**

   $S_1 =$

   $S_2 =$

   $S_3 =$

   $S_4 =$

   $S_n =$

   (c) Find the sum $S$ of the original series, if it exists.

3. Find the sum, if it exists, of the geometric series $\sum_{n=1}^{\infty} \frac{2 \cdot 3^n}{4^n}$

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|---|---|---|
| Series Tests: | | |
| 1. $n$th Term Test | 3. Ratio Test | 5. Integral Test |
| 2. Special Series Tests: | 4. Limit Comparison Test | 6. Alternating Series Test |
| a. Geometric Series | | |
| b. $p$-series | | |

4. Determine if the following series **converge** or **diverge**. State the test you are using and explain your work.

(a) $\sum_{n=1}^{\infty} \frac{n+1}{2n-1}$

(b) $\sum_{n=1}^{\infty} \frac{1}{[(n+1)\ln(n+1)][\ln(\ln(n+1))]}$

(c) $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \cdots$

5. Determine whether the **alternating series** $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\ln(n+1)}{n+1}$ is **absolutely convergent, conditionally convergent, or divergent.** Show all work.

6. Find the **interval of convergence** for the power series $\sum_{n=1}^{\infty}\frac{(x-2)^{n+1}}{(n+1)\cdot 4^{n+1}}$. (Don't forget to check endpoints.)

7. Use the **definition of the Taylor Series** and write the first 3 non-zero terms for $f(x) = e^{(-2x)}$ centered at $c = 0$. Then write the series using sigma notation.

8. Use the Taylor Series $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, and write the first 4 terms and the series in sigma notation for:

$\cosh x = \frac{1}{2}(e^x + e^{-x}) =$

9. Use the Taylor Series $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$, and write the first 4 terms and the series in sigma notation for:

$\int \cos(\sqrt{x})\, dx =$