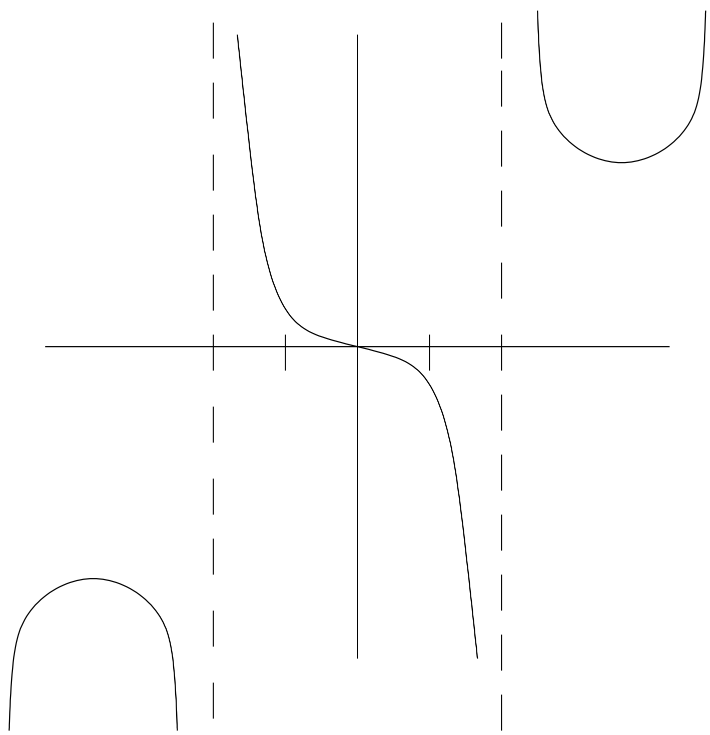
Solutions to Assignment #9

Worksheet #9 (6 pts)

Asymptotes: $x = -2$, $x = 2$
Intercepts: $(0, 0)$
Critical #'s : $x = 0, 2\sqrt{3}, -2\sqrt{3}, -2, 2$
Increasing: $(-\infty, -2\sqrt{3}), (2\sqrt{3}, \infty)$
Decreasing: $(-2\sqrt{3}, -2), (-2, 2), (2, 2\sqrt{3})$
Maximum: $(-2\sqrt{3}, -3\sqrt{3})$ Minimum: $(2\sqrt{3}, 3\sqrt{3})$
Possible inflection points: $x = 0, -2, 2$
Concave up: $(-2, 0), (2, \infty)$
Concave down: $(-\infty, -2), (0, 2)$
Inflection points: $(0, 0)$



4.3 #16 (5 pts)

(a)

$$\begin{aligned} f(x) &= x^4(x-1)^3 \\ f'(x) &= 4x^3(x-1)^3 + 3x^4(x-1)^2 \\ &= x^3(x-1)^2(4(x-1)+3x) \\ &= x^3(x-1)^2(4x-4+3x) \\ &= x^3(x-1)^2(7x-4) \end{aligned}$$

Therefore our critical numbers are

$$c_1 = 0, 1, \frac{4}{7}$$

(b)

$$\begin{aligned} f''(x) &= 3x^2(x-1)^2(7x-4) + 2x^3(x-1)(7x-4) + 7x^3(x-1)^2 \\ \text{So } f''(0) &= 0 \\ f''(1) &= 0 \\ \text{and } f''(\frac{4}{7}) &= 7(\frac{4}{7})^3(\frac{4}{7}-1)^2 > 0 \\ &\Rightarrow \frac{4}{7} \text{ is a local minimum} \end{aligned}$$

The test is inconclusive for the other two critical numbers.

(c) On the interval $(-\infty, 0)$ $f'(x) > 0$, while on the interval $(0, \frac{4}{7})$ $f'(x) < 0$, and on the interval $(\frac{4}{7}, 1)$ $f'(x) > 0$ and finally on the interval $(1, \infty)$ $f'(x) > 0$. So all this implies that $x = 0$ is a local maximum, $x = \frac{4}{7}$ is a local minimum, and $x = 1$ is neither.

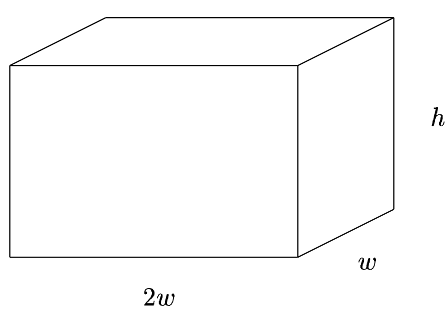
4.3 #48 (5 pts)

Let $v(t)$ be the velocity of the car at a given time t in hours. Then the acceleration of the car is the function $v'(t)$. Now we know that $v(t)$ is continuous, and that $v(2:00) = 30$, and that $v(2:10) = 50$. So by the mean value theorem we know that there exists some time say t' between $2:00$, and $2:10$ such that

$$\begin{aligned} v'(t') &= \frac{v(2:10) - v(2:00)}{2:10 - 2:00} \\ &= \frac{50-30}{\frac{1}{6}} \\ &= (20)(6) \\ &= 120\frac{\text{mi}}{\text{hr}^2} \end{aligned}$$

Which is precisely what we needed to show.

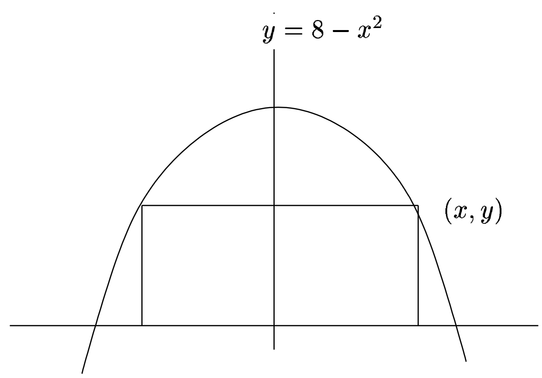
4.6 #12 (5 pts)



$$\begin{aligned} v &= (2w)(w)(h) \\ &= 2w^2h = 10 \\ h &= \frac{5}{w^2} \end{aligned}$$

$$
\begin{aligned}
C &= 10(2w^2) + 6(4wh + 2wh) \\
&= 20w^2 + 36wh \\
&= 20w^2 + \frac{180}{w} \\
C' &= 40w - \frac{180}{w^2} \\
40w - \frac{180}{w^2} &= 0 \\
40w^3 - 180 &= 0 \\
w &= \sqrt[3]{\frac{9}{2}} = 1.65 \\
&\Rightarrow C = \$163.54
\end{aligned}
$$

4.6 #16 (4 pts)



$$
\begin{aligned}
A &= 2xy \\
&= 2x(8 - x^2) \\
&= 16x - 2x^3 \\
A' &= 16 - 6x^2 \\
16 - 6x^2 &= 0 \\
x^2 &= \frac{8}{3} \\
x &= \sqrt{\frac{8}{3}} = 1.633 \\
y &= 5.333
\end{aligned}
$$