Solutions to Assignment #6

1.7 #10 (3 pts)

(a)

$$x = 4\cos(\theta) \qquad y = 5\sin(\theta)$$

$$\cos(\theta) = \frac{x}{4} \qquad \sin(\theta) = \frac{y}{5}$$

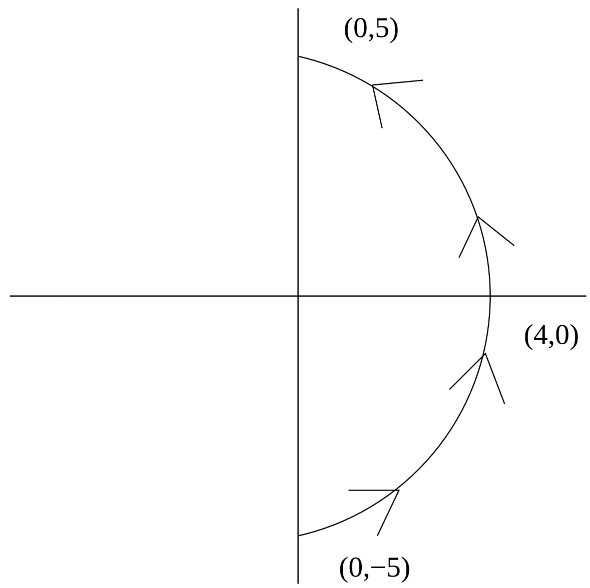
$$\text{So, } 1 = \cos^2(\theta) + \sin^2(\theta) = (\frac{x}{4})^2 + (\frac{y}{5})^2$$

giving us the Cartesian equation

$$(\frac{x}{4})^2 + (\frac{y}{5})^2 = 1$$

(b)

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
x	0	$2\sqrt{2}$	4	$2\sqrt{2}$	0
y	-5	$-\frac{5\sqrt{2}}{2}$	0	$\frac{5\sqrt{2}}{2}$	5



3.5 #42 (4 pts)

(a)

$$\begin{aligned} h'(x) &= f'(f(x))f'(x) \\ h'(2) &= f'(f(2))f'(2) \\ &= f'(1)f'(2) \\ &\approx (-1)(-1) = 1 \end{aligned}$$

(b)

$$\begin{aligned} g'(x) &= f'(x^2)(2x) \\ g'(2) &= f'(4)(4) \\ &\approx (2)(4) = 8 \end{aligned}$$

3.5 #68 (7 pts)

(a)

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{r\sin(\theta)}{r - r\cos(\theta)} = \frac{\sin(\theta)}{1-\cos(\theta)}$$

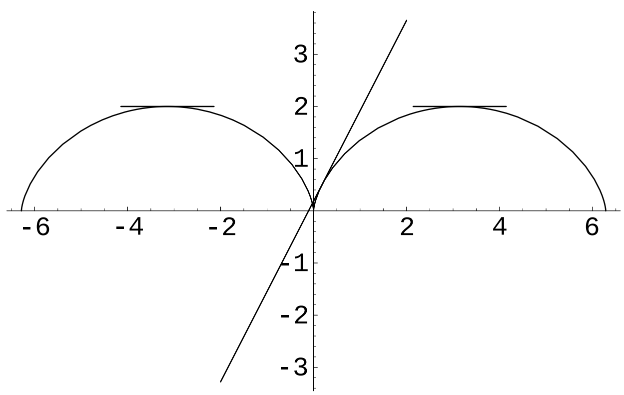
$$\text{When } \theta = \frac{\pi}{3} \qquad \frac{dy}{dx} = \sqrt{3}$$

and the corresponding point at $\theta = \frac{\pi}{3}$ is $(\frac{r\pi}{3} - \frac{r\sqrt{3}}{2}, \frac{r}{2})$
So the equation of the tangent line is

$$y - \frac{r}{2} = \sqrt{3}(x - (\frac{r\pi}{3} - \frac{r\sqrt{3}}{2}))$$

(b) Horizontal tangent $\Rightarrow \sin(\theta) = 0$, and $\cos(\theta) \neq 1$.
This occurs at $\theta = -\pi$, and $\theta = \pi$
Vertical tangent $\Rightarrow \cos(\theta) = 1$, and $\sin(\theta) \neq 0$, and this does not occur.

(c)



3.6 #6 (4 pts)

$$\begin{aligned}
\frac{d}{dx}[y^5 + x^2y^3] &= \frac{d}{dx}[1 + ye^{x^2}] \\
5y^4y' + x^2 3y^2y' + y^3 2x &= ye^{x^2}2x + e^{x^2}y' \\
5y^4y' + 3x^2y^2y' - y'e^{x^2} &= 2xye^{x^2} - 2xy^3 \\
y'(5y^4 + 3x^2y^2 - e^{x^2}) &= 2xye^{x^2} - 2xy^3 \\
y' &= \frac{2xye^{x^2} - 2xy^3}{5y^4 + 3x^2y^2 - e^{x^2}}
\end{aligned}$$

3.6 #20 (7 pts)

(a)

$$\begin{aligned}
2yy' &= 3x^2 + 6x \\
y' &= \frac{3x^2 + 6x}{2y}
\end{aligned}$$

So at $(1, -2)$ $y' = -\frac{9}{4}$
So the equation of the line is

$$y + 2 = -\frac{9}{4}(x - 1)$$

(b)

$$\begin{aligned} \text{Horizontal tangent } &\Rightarrow 3x^2 + 6x = 0 \\ &\Rightarrow 3x(x+2) = 0 \\ &\Rightarrow x = 0 \text{ or } x = -2 \end{aligned}$$

But at $x = 0$, $y = 0$, so $\frac{dy}{dx}$ does not exist.
At $x = -2$, $y = -2$ or $y = 2$ so the points are $(-2, -2)$ and $(-2, 2)$.

(c)