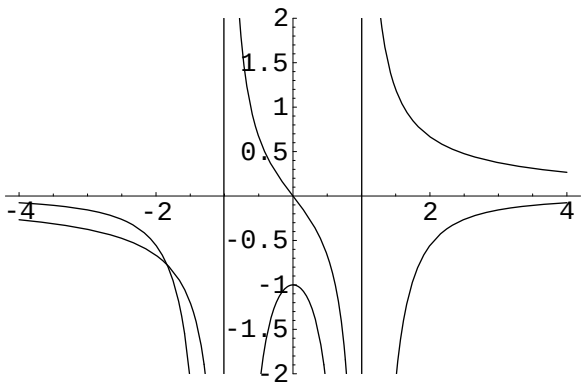
Solutions to Assignment #5

3.2 #24 (3 pts)

(a)

$$f'(x) = \frac{(x^2-1)\cdot 1 - x(2x)}{(x^2-1)^2}$$
$$= \frac{-x^2-1}{(x^2-1)^2}$$

(b)



3.4 #8 (2 pts)

$$y' = \frac{(1+\cos x)\cos x - (\sin x)\cdot(-\sin x)}{(1+\cos x)^2}$$
$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$$
$$= \frac{1+\cos x}{(1+\cos x)^2}$$
$$= \frac{1}{1+\cos x}$$

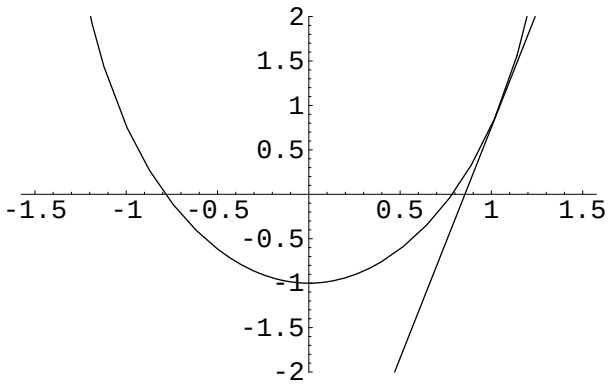
3.4 #20 (4 pts)

(a)

$$y' = \sec x \tan x + 2\sin x$$
$$y'(\frac{\pi}{3}) = (\sec(\frac{\pi}{3})\cdot\tan(\frac{\pi}{3})) + 2\sin(\frac{\pi}{3})$$
$$= 2\sqrt{3} + 2(\frac{\sqrt{3}}{2})$$
$$= 3\sqrt{3}$$

Equation: $y - 1 = 3\sqrt{3}(x - \frac{\pi}{3})$

(b)



3.4 #24 (3 pts)

$$\begin{aligned} f'(x) &= (\sec x)(\tan x) \\ f''(x) &= (\sec x)\cdot(\sec^2 x) + (\tan x)((\sec x)(\tan x)) \\ &= \sec^3 x + (\sec x)(\tan^2 x) \\ f''(\frac{\pi}{4}) &= (\sqrt{2})^3 + \sqrt{2}(1) \\ &= 2\sqrt{2} + \sqrt{2} \\ &= 3\sqrt{2} \end{aligned}$$

3.4 #28 (3 pts)

$$\begin{aligned} f'(x) &= 1 - \cos x \\ f''(x) &= \sin x \\ f \text{ is concave up } &\Rightarrow f''(x) > 0 \\ &\Rightarrow \sin x > 0 \\ &\Rightarrow x \text{ is in } (0, \pi) \end{aligned}$$

3.5 #10 (3 pts)

$$\begin{aligned} f(t) &= (1 + \tan t)^{\frac{1}{3}} \\ f'(t) &= \frac{1}{3}(1 + \tan t)^{-\frac{2}{3}} * \sec^2 t \\ &= \frac{\sec^2 t}{3\sqrt[3]{(1 + \tan t)^2}} \end{aligned}$$

3.5 #18 (3 pts)

$$\begin{aligned} g'(t) &= (6t^2 + 5)^3(4)(t^3 - 7)^3(3t^2) + (t^3 - 7)^4(3)(6t^2 + 5)^2(12t) \\ &= 12t^2(6t^2 + 5)^3(t^3 - 7)^3 + 36t(6t^2 + 5)^2(t^3 - 7)^4 \\ &= 12t(6t^2 + 5)^2(t^3 - 7)^3(t(6t^2 + 5) + 3(t^3 - 7)) \\ &= 12t(6t^2 + 5)^2(t^3 - 7)^3(9t^3 + 5t - 21) \end{aligned}$$

3.5 #52 (4 pts)

$$y = e^{rx},\ y' = re^{rx},\ y'' = r^2 e^{rx}$$

$$
\begin{aligned}
&y''+5y'-6y=0\\
&\quad\Rightarrow r^2\mathrm{e}^{rx}+5r\mathrm{e}^{rx}-6\mathrm{e}^{rx}=0\\
&\quad\Rightarrow \mathrm{e}^{rx}(r^2+5r-6)=0\\
&\quad\Rightarrow r^2+5r-6=0\\
&\quad\Rightarrow (r+6)(r-1)=0\\
&\quad\Rightarrow r=-6,\, r=1
\end{aligned}
$$