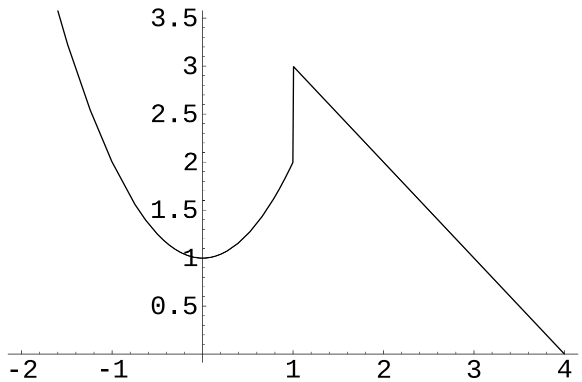
Solutions to Assignment #2

2.4 # 16 (2 pts)

$$f(x) = \begin{cases} 1 + x^2 & \text{if } x < 1 \\ 4 - x & \text{if } x \geq 1 \end{cases}$$



$f(x)$ is discontinuous at $x = 1$ because the $\lim_{x\to 1} f(x)$ does not exist, because $\lim_{x\to 1^+} f(x) \neq \lim_{x\to 1^-} f(x)$.

2.4 # 36 (3 pts)

Let $f(x) = x^2 - \sqrt{x+1}$. Note that $f(x)$ is continuous on the interval $[1, 2]$. Also, $f(1) = (1)^2 - \sqrt{1+1} = 1 - \sqrt{2} < 0$, and $f(2) = (2)^2 - \sqrt{2+1} = 4 - \sqrt{3} > 0$. So let $N = 0$ so that $f(1) < N < f(2)$. Then since $f(x)$ is continuous on the interval $[1, 2]$ then by the intermediate value theorem, there exists $c \in (1, 2)$ such that $f(c) = N = 0$. Therefore $f(x) = x^2 - \sqrt{x+1}$ has a root in the interval $(1, 2)$.

2.4 #45 (3 pts)

A number that is one more than its cube satisfies the equation $x = x^3 + 1$. So let $f(x) = x^3 + 1 - x$. Then notice that $f(x)$ is continuous on the interval $[-2, 2]$, $f(-2) = (-2)^3 + 1 - (-2) = -5 < 0$, and $f(2) = (2)^3 + 1 - (2) = 7 > 0$. So let $N = 0$ then by the intermediate value theorem, there exists $c \in (-2, 2)$ such that $f(c) = N = 0$. Therefore $f(x) = x^3 + 1 - x$ has a root in the interval $(-2, 2)$. So there is a number that is one more than its cube.

2.5 #4 (3 pts)

(a)

$$\lim_{x\to\infty} g(x) = 2$$

(b)

$$\lim_{x\to-\infty} g(x) = -2$$

(c)

$$\lim_{x\to 3} g(x) = \infty$$

(d)

$$\lim_{x\to 0} g(x) = -\infty$$

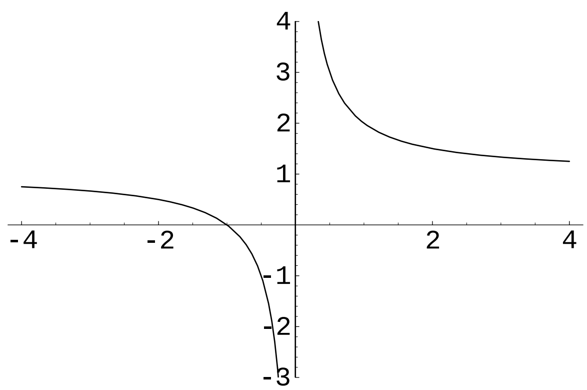
(e)

$$\lim_{x\to 2^+} g(x) = -\infty$$

(f)

$$\begin{aligned} x &= -2 \\ x &= 0 \\ x &= 3 \\ y &= -2 \\ y &= 2 \end{aligned}$$

2.5 #6 (2 pts)



2.5 #18 (1 pt)

$$\lim_{x\to\infty} \frac{3x+5}{x-4} = 3$$

2.5 #20 (1 pt)

$$\lim_{x\to\infty} \frac{t^2+2}{t^3+t^2-1} = 0$$

2.6 #8 (4 pts)

Using our defintion for the derivative

$$\begin{aligned} m &= \lim_{h\to 0} \frac{2(-1+h)^3 - 5(-1+h) - (2(-1)^3 - 5(-1))}{h} \\ &= \lim_{h\to 0} \frac{2(-1+h+2h-2h^2-h^2+h^3)+5-5h-(-2+5)}{h} \\ &= \lim_{h\to 0} \frac{h(1-6h+2h^2)}{h} \\ &= \lim_{h\to 0} 1-6h+2h^2 \\ &= 1 \end{aligned}$$

Thus the equation of the tangent line is $y-3=(x+1)$

2.6 #16 (6 pts)

(a)

$$v(1) = \lim_{h\to 0} \frac{H(1+h)-H(1)}{h}$$
$$= \lim_{h\to 0} \frac{(58+58h-.83-1.66h-.83h^2)-(57.17)}{h}$$
$$= \lim_{h\to 0}(56.34-.83h)$$
$$= 56.34\frac{\text{m}}{\text{s}}$$

(b)

$$v(a) = \lim_{h\to 0} \frac{H(a+h)-H(a)}{h}$$
$$= \lim_{h\to 0} \frac{(58a+58h-.83a^2-1.66ah-.83h^2)-(58a-.83a^2)}{h}$$
$$= \lim_{h\to 0}(58-1.66a-.83h)$$
$$= 58-1.66a\frac{\text{m}}{\text{s}}$$

(c)

$$58t-.83t^2=0$$

solve for t, to get

$$t \approx 69.9\text{s}$$

(d)

$$v(69.9) = 58-1.66(69.9) = -58\frac{\text{m}}{\text{s}}$$