Math 1401 — Maxima and Minima Definitions and Theorems

1. **Definition:** Let a function f be defined on [a,b], and let c be in [a,b].
   a. f(c) is the **absolute maximum value** of f on [a,b] if $f(c) \geq f(x)$ for every x in [a,b].
   b. f(c) is the **absolute minimum value** of f on [a,b] if $f(c) \leq f(x)$ for every x in [a,b].

2. **Extreme Value Theorem:** If f is continuous on [a,b], then f takes on an **absolute maximum** and an **absolute minimum** value at least once in [a,b].

3. **Definition:** Let c be a number in the domain of f.
   a. f(c) is a **local maximum** of f if there exists an (a,b) containing c such that $f(c) \geq f(x)$ for all x in (a,b).
   b. (c) is a **local minimum** of f if there exists an (a,b) containing c such that $f(c) \leq f(x)$ for all x in (a,b).

4. **Theorem:** If a function f has a **local extremum** (maximum or minimum) at a number c in an open interval, then either $f'(c)=0$ or $f'(c)$ does not exist.

5. **Definition:** A number c in the domain of f is a **critical number** of f if either $f'(c)=0$ or $f'(c)$ does not exist.

6. **Test For Absolute Extrema:** To find **absolute extrema on a closed interval [a,b],**
   a. Compute $f'(x)$ and find all critical numbers c in (a,b)
   b. Calculate f(a), f(b), and f(c) for each critical number c.
   c. The largest of these values is the **absolute maximum** and the smallest of these values is the **absolute minimum** on [a,b].

7. **Theorem:** Let f be a continuous and differentiable function on [a,b].
   a. If $f'(x)>0$ for every x in (a,b), then f is **increasing** on [a,b].
   b. If $f'(x)<0$ for every x in (a,b), then f is **decreasing** on [a,b].

8. **The First Derivative Test:** Let c be in (a,b) and let c be a critical number of a continuous function f.
   a. If f' changes from positive to negative at c, then f has a **local maximum** at c.
   b. If f' changes from negative to positive at c, then f has a **local minimum** at c.
   c. If f' does not change sign at c, then f has no local maximum or minimum at c.

9. **Theorem:** Let f be a continuous function on [a,b].
   a. If $f''(x)>0$ for every x in (a,b), then f is **concave up** on [a,b].
   b. If $f''(x)<0$ for every x in (a,b), then f is **concave down** on [a,b].

10. **The Second Derivative Test:** Let c be in (a,b) and let $f'(c)=0$.
    a. If $f''(c)<0$, then f has a **local maximum** at c.
    b. If $f''(c)>0$, then f has a **local minimum** at c.
    c. If $f''(c)=0$, then this test does not apply.