

# Probability (Math 4810/5210)

## SAMPLE EXAM 1.

Instructions. You are supposed to score at least 80% at each level in order to be eligible to get to the next level. In order to get an A you must give the correct solution of the A-level problem.

- **C-level questions.**

Circle the correct answer.

1. An experiment consists in first rolling a six-sided die and then flipping a two-sided coin. Find the number of outcomes of the experiment.
  - A. 6
  - B. 2
  - C. 12
  - D. 8
2. Darts are thrown at random at a square target of size 40cm by 40cm. What is the probability that a dart will hit a square of size 20cm by 20cm inside the target ?
  - A.  $1/2$
  - B.  $1/3$
  - C.  $1/4$
  - D. None of the above
3. Suppose the probability of event  $A$  is  $1/2$  and the probability of event  $B$  is  $3/4$ . Given that the events  $A$  and  $B$  are independent, what is the probability that  $A$  and  $B$  occur together ?
  - A. 0
  - B. 1.25
  - C.  $3/8$
  - D.  $3/4$
4. An experiment of rolling a 3-sided die has three outcomes: 1,2, and 3. Outcome 3 is three times more likely to occur than outcome 1 and 1.5 times more likely to occur than outcome 2. What is the probability of outcome 1 ?
  - A. 1
  - B.  $1/6$
  - C.  $1/5$
  - D. None of the above
5. The students in a class of 3 boys and 7 girls are ranked according to their results on a test. Assuming that all rankings are equally likely, find the probability that the highest ranked is a girl.
  - A.  $1/2$
  - B. .7
  - C. .4
  - D. None of the above

• **B-level questions**

1. In how many ways can a committee of 2 men and 3 women be chosen out of 6 men and 7 women ?
2. A stockbroker is researching 13 independent stocks. An investment in each stock will either make money or lose money. The probability that each stock will make money is  $5/8$ . What is the probability that exactly 10 of the stocks make money?
3. Suppose  $X$  and  $Y$  are independent random variables which are uniformly distributed on  $[0, 1]$ . Given that  $X + 2Y > 1$ , find the probability that  $\max(X, Y) \leq 1/2$ .
4. The loss due to a fire in a commercial building is modeled by a random variable  $X$  with density function

$$f(x) = \begin{cases} 0.005(20 - x), & \text{for } 0 \leq x \leq 20, \\ 0, & \text{otherwise} \end{cases}$$

Given that a fire loss exceeds 8, what is the probability that it exceeds 16 ?

5. A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of the disease 0.5% of the time when the disease is not present. One percent of the population actually has the disease. Calculate the probability that a person has the disease given that the test indicates the presence of the disease.

• **A-level questions**

1. A doctor is studying the relationship between blood pressure and heartbeat abnormalities in her patients. She tests a random sample of her patients and notes their blood pressures (high, low, or normal) and their heartbeats (regular or irregular). She finds that:
  - (i) 14% have high blood pressure.
  - (ii) 22% have low blood pressure.
  - (iii) 15% have an irregular heartbeat.
  - (iv) Of those with an irregular heartbeat, one-third have high blood pressure.
  - (v) Of those with normal blood pressure, one-eighth have an irregular heartbeat.What portion of the patients selected have a regular heartbeat and low blood pressure?
2. A device runs until either of two components fails, at which point the device stops running. The joint density function of the lifetimes of the two components, both measured in hours, is

$$f(x, y) = \frac{x + y}{27} \text{ for } 0 \leq x \leq 3 \text{ and } 0 \leq y \leq 3.$$

Calculate the probability that the device fails during its first hour of operation.

3. You put \$5.00 into one of 6 pockets in your jeans, but you cannot remember which pocket. After checking two pockets without success, what is the probability that the \$5.00 will be in the next pocket you check?