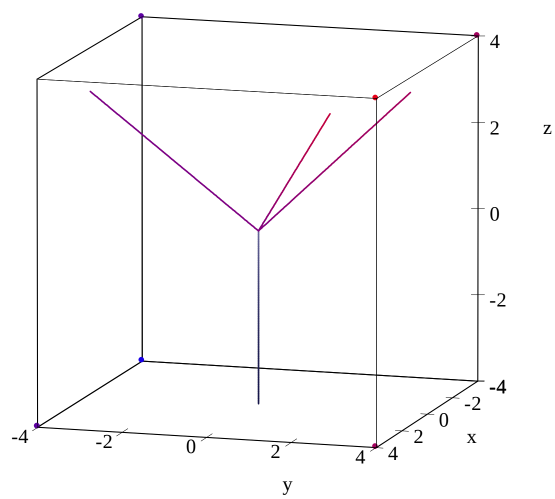
The Hanging Package (Engineer-Style) – MATH 2421
Summer 2005

Instructions. The problem is equivalent to a 3×3 system of equations (3 equations in 3 unknowns) which has a unique solution. We will use Derive5 to solve it.

(#1) Suppose we have a hanging package at the origin $O\,(0,0,0)$ with three support lines.

The support lines end at $P\,(0,-4,3)$, $Q\,(1,2,3)$, and $R\,(-2,3,3)$.

The weight vector points downward and has magnitude 50 N. Thus, we have $\mathbf{w} = \langle 0,0,-50\rangle$.



The weight vector is clearly NOT drawn to scale.

(#2) We know that the three tension vectors run parallel (within) to the support lines. It turns out that these forces all point upward.

According to Isaac Newton, the sum of the three tension vectors and the downward weight vector should be the zero vector, assuming that the package is at rest (not accelerating in any direction).

Let $a * \overrightarrow{OP}$ = the tension in the $\overrightarrow{OP}$ direction.

Let $b * \overrightarrow{OQ}$ = the tension in the $\overrightarrow{OQ}$ direction.

Let $c * \overrightarrow{OR}$ = the tension in the $\overrightarrow{OR}$ direction.

Do you see our strategy? Rather than calculating unit direction vectors and accumulating a bunch of square roots, we sum up the integer components and the resulting values of a, b, and c end up being rational numbers.

It IS true that the magnitudes of the final tension vectors are irrational, but it is certainly easier to deal with that at the end of the process!

(#3) Our vector equation is:

$$a\langle 0,-4,3\rangle + b\langle 1,2,3\rangle + c\langle -2,3,3\rangle + \langle 0,0,-50\rangle = \langle 0,0,0\rangle.$$

When we equate the components, we obtain three equations in three unknowns.

$$\begin{aligned} 0a + 1b - 2c &= 0 \\ -4a + 2b + 3c &= 0 \\ 3a + 3b + 3c &= 50 \end{aligned}$$

I moved the (-50) in the third equation to the right side in order to save space.

As before, in Derive5, select the "Solve" pull-down menu and then select "System".

We have three equations. Type them in. Click the cursor in the "Solution Variables" box. Click on "Solve".

This gives us $a = \dfrac{350}{57}$, $b = \dfrac{400}{57}$, and $c = \dfrac{200}{57}$.

Thus, the tension vector in the OP direction is $\dfrac{350}{57}\langle 0,-4,3\rangle$ and its magnitude is

$$\left(\frac{350}{57}\right)(5) \doteq 30.7 \text{ N}.$$

Verify that the magnitudes of the other tension vectors are approximately 26.3 N and 16.5 N.