Solutions to Quiz #01 – MATH 2421
Summer 2006

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1. Let $\mathbf{v} = \langle 2,\ -3,\ 6 \rangle$.

   (a) Find the associated unit vector (same direction).
   Divide $\mathbf{v}$ by its (nonzero) magnitude.

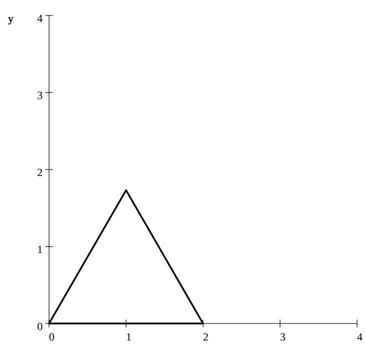
   $$\mathbf{u} = \frac{\langle 2,\ -3,\ 6 \rangle}{\sqrt{2^2 + (-3)^2 + 6^2}} = \frac{\langle 2,\ -3,\ 6 \rangle}{7} = \left\langle \frac{2}{7},\ -\frac{3}{7},\ \frac{6}{7} \right\rangle .$$

   (b) Now resize the answer to part (a) so that the new vector has magnitude 13.
   Just multiply by the scalar 13.

   $$\mathbf{w} = 13 \left\langle \frac{2}{7},\ -\frac{3}{7},\ \frac{6}{7} \right\rangle = \left\langle \frac{26}{7},\ -\frac{39}{7},\ \frac{78}{7} \right\rangle .$$

2. Sketchapalooza

   (a) On the set of axes below, sketch in points P, Q, and R such that they form an EQUILATERAL triangle.
   Place P at the origin and then you decide where to put Q and R.



As long as your figure had sides of equal length, the orientation of the triangle did not matter.
Remember that the angles in an equilateral triangle must all be 60°

It turns out that you were better off NOT giving the coordinates of the vertices. You didn't need them!
If you gave incorrect coordinates, then I deducted points because these would clearly form a triangle which was NOT equilateral.

In my example, each of my sides had length 2. Here are my points:
$P\,(0,0)$, $Q\,(2,0)$, $R\left(1, \sqrt{3}\right)$.

   (b) Now place arrowheads at the appropriate places and form the vectors $\overrightarrow{PQ}$, $\overrightarrow{QR}$, and $\overrightarrow{RP}$.
   Since my software won't print arrowheads, you should label my diagram with P, Q, and R and place arrowheads running from P to Q, from Q to R, and from R to P.
   By coordinate subtraction, the component forms of my example vectors are

$$\begin{aligned} \overrightarrow{PQ} &= \langle 2,\ 0 \rangle \\ \overrightarrow{QR} &= \left\langle -1,\ \sqrt{3} \right\rangle \\ \overrightarrow{RP} &= \left\langle -1,\ -\sqrt{3} \right\rangle . \end{aligned}$$

(c) Find the sum:

$$\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RP} = ???$$

Clearly, if I add the components of my three example vectors, I obtain

$$\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RP} = \langle 0,\ 0 \rangle = \mathbf{0}.$$

I did NOT need the components to know this. If we let P, Q, and R be the vertices of *any* triangle, then the three vectors, $\overrightarrow{PQ}$, $\overrightarrow{QR}$, and $\overrightarrow{RP}$, always form a closed path and since the resultant connects the first tail (P) to the last head (also P), the resultant must be the zero vector!

3. Miscellaneous.

(a) With what angle $\theta \quad (0 \leq \theta < 2\pi)$ do we associate the unit direction vector

$$\left\langle \frac{1}{2},\ -\frac{\sqrt{3}}{2} \right\rangle = \langle \cos(\theta),\ \sin(\theta) \rangle ?$$

This angle must be in Quadrant IV.

$$\tan(\theta) = \frac{\textit{vertical component}}{\textit{horizontal component}} = -\sqrt{3}.$$

Normally, we have $\tan^{-1}\left(-\sqrt{3}\right) = -\dfrac{\pi}{3}$, but we want a positive angle, so we will add 2π.

The correct answer is $\theta = \dfrac{5\pi}{3} = 300°$.

(b) What is the Cartesian equation of the plane which is parallel to the xy-plane, but is located 5 units below the xy-plane?

It must be $z = \textit{a constant}$. The correct answer is $z = -5$.

(c) Find the center and radius of this sphere.

$$x^2 + y^2 - 6y + z^2 + 12z = -29$$

Complete the square twice.

$$\begin{aligned} x^2 + y^2 - 6y + (-3)^2 + z^2 + 12z + 6^2 &= -29 + 9 + 36 \\ (x-0)^2 + (y-3)^2 + (z+6)^2 &= 16 = R^2. \end{aligned}$$

The center is located at $C(0,\ 3,\ -6)$ and the radius is 4.