Solutions to Quiz #08 – MATH 2421

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1. The surface area differential is

$$dS = \sqrt{1 + (f_x)^2 + (f_y)^2}\ dA.$$

Suppose we have the upper cone $z = 2\sqrt{x^2 + y^2}$. Find the surface area above the region in the xy-plane R: $x^2 + y^2 \leq 4$.

$r\ dr\ d\theta$

r: $0 \to 2$

θ: $0 \to 2\pi$

$$\begin{aligned} dS &= \sqrt{1 + \left(\frac{2x}{\sqrt{x^2+y^2}}\right)^2 + \left(\frac{2y}{\sqrt{x^2+y^2}}\right)^2} \\ &= \sqrt{1 + \frac{4x^2 + 4y^2}{x^2 + y^2}} = \sqrt{1+4} = \sqrt{5}. \end{aligned}$$

The surface area is

$$\int_0^{2\pi} \int_0^2 \sqrt{5}\ r\ dr\ d\theta = \sqrt{5} * (Area\ of\ R) = 4\sqrt{5}\pi.$$

2. We have a solid Q bounded by the plane $x = y + 2$ and the hemisphere $x = -\sqrt{2 - y^2 - z^2}$.

The solid is also bounded by the planes $y = -1$, $y = 1$, $z = -1$, and $z = 1$.

Suppose the density function is $\sigma(x, y, z) = x^2 + 2z^2$.

The back surface is $x = -\sqrt{2 - y^2 - z^2}$.

The front surface is $x = y + 2$.

The "sides" of the surface are the four planes.

Thus, we choose:

$dx\ dz\ dy$

x: $-\sqrt{2 - y^2 - z^2} \to y + 2$

z: $-1 \to 1$

y: $-1 \to 1$.

$$m = \int_{-1}^{1} \int_{-1}^{1} \int_{-\sqrt{2-y^2-z^2}}^{y+2} \left(x^2 + 2z^2\right)\ dx\ dz\ dy$$

3. Suppose we have the surface of revolution

$$z = \ln\left(1 + x^2 + y^2\right) = \ln\left(1 + r^2\right).$$

Hint: This is easy to transform into cylindrical coordinates. [The rz-plane generating curve is given below as an added hint.]

The solid Q traps the volume *beneath* this surface, above the circular region $x^2+y^2 \leq 1$ ONLY IN QUADRANT I.

The density function is $\sigma(x, y, z) = x + y = r\cos(\theta) + r\sin(\theta)$.

Set up but DO NOT EVALUATE the CYLINDRICAL COORDINATES TRIPLE INTEGRAL which represents the mass.

$dz\ r\ dr\ d\theta$

z: $0 \to \ln\left(1 + r^2\right)$

r: $0 \to 1$

θ: $0 \to \pi/2$ [Quadrant I only]

$$m = \int_0^{\pi/2} \int_0^1 \int_0^{\ln\left(1+r^2\right)} \left(r\cos(\theta) + r\sin(\theta)\right)\ dz\ r\ dr\ d\theta$$