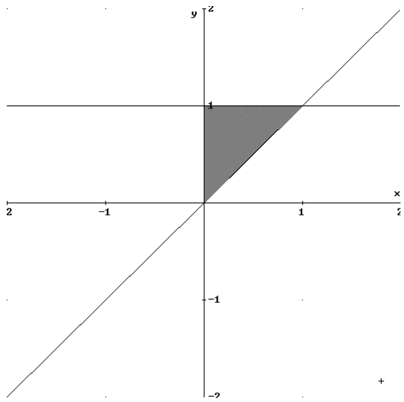
**Solutions to Quiz #07 – MATH 2421**

Puhalskii

1. Reverse the order of integration and evaluate the double integral.

$$\int_0^1 \int_x^1 \frac{2}{1+(y^2)^2} dy\ dx = ???$$




$y$: $x \to 1$
$x$: $0 \to 1$

Horizontal first...

$x$: $0 \to y$

$y$: $0 \to 1$.

Thus, we have

$$\int_0^1 \int_0^y \frac{2}{1+(y^2)^2} dx\ dy = ???$$

Inner:

$$\frac{2}{1+(y^2)^2} \int_0^y dx = \frac{2y}{1+(y^2)^2}$$

Outer: $u = y^2$, $du = 2y\ dy$.

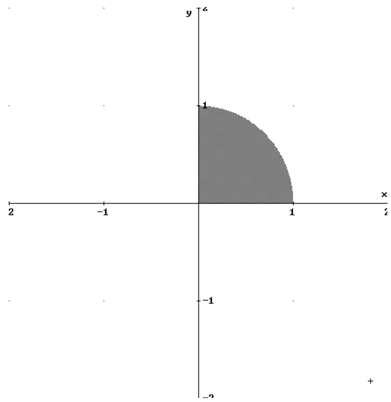
$$\int_0^1 \frac{2y}{1+(y^2)^2} dy \quad \Rightarrow \quad \int \frac{1}{1+u^2} du = \tan^{-1}(u)$$

$$= \quad \left[\tan^{-1}\left(y^2\right)\right]_0^1 = \tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4}.$$

2. The polar region $R$ is depicted below. Evaluate this double integral:

$$\iint\limits_R y^2 \; dA = ??$$

$R$: $x^2 + y^2 \leq 1$, Quadrant I only.




Hints: The polar area differential is $dA = r \; dr \; d\theta$.
$x = r\cos(\theta) \quad y = r\sin(\theta)$

$r$: $0 \to 1$

$\theta$: $0 \to \pi/2$.

The integrand becomes $y^2 = (r\sin(\theta))^2 = r^2\sin^2(\theta)$.
Thus, our polar double integral is

$$\begin{aligned}
\int_0^{\pi/2}\int_0^1 r^2\sin^2(\theta) \; r \; dr \; d\theta &= \left(\int_0^1 r^3 dr\right)\left(\int_0^{\pi/2} \sin^2(\theta)\, d\theta\right) \\
&= \frac{1}{4}\left[\frac{\theta}{2} - \frac{\sin(2\theta)}{4}\right]_0^{\pi/2} \\
&= \frac{1}{4}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{16}.
\end{aligned}$$