Solutions to Quiz #05 – MATH 2421

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1. Suppose we have the following:

$$z = f(x, y) = \frac{y}{x^2}$$

   (a) Evaluate $f(2,\ 20)$.

$$f(2,\ 20) = \frac{20}{2^2} = 5.$$

   (b) Evaluate the total differential

$$dz = f_x dx + f_y dy = \left(-\frac{2y}{x^3}\right) dx + \left(\frac{1}{x^2}\right) dy$$

   (c) Estimate the value of Δz using dz when x moves from 2.00 to 2.04 AND y moves from 20.00 to 19.98.

   We have $dx = 2.04 - 2.00 = +0.04$ and $dy = 19.98 - 20.00 = -0.02$.

$$\Delta z \approx dz = \left(-\frac{2(20)}{2^3}\right)(0.04) + \left(\frac{1}{2^2}\right)(-0.02) = -0.205.$$

   Since we have a calculator, we can check this.

$$\Delta z = f(2.04,\ 19.98) - f(2,\ 20) = \frac{19.98}{(2.04)^2} - 5 = -0.199.$$

   It's reasonably close.

(#2) Suppose $w = f(x, y)$ and both x and y depend on s and t:

$$\begin{aligned} x &= x(s, t) \\ y &= y(s, t) \end{aligned}$$

Also, suppose s and t depend on θ:

$$\begin{aligned} s &= s(\theta) \\ t &= t(\theta) \end{aligned}$$

Thus, after substituting, w ultimately depends on θ.

Write down the Multivariable Chain Rule for $\dfrac{\partial w}{\partial \theta}$.

$$\begin{array}{ccccccc} & & & f & & & \\ & & \swarrow & & \searrow & & \\ & x & & & & y & \\ \swarrow & \downarrow & & & & \downarrow & \searrow \\ s & t & & & & s & t \\ \downarrow & \downarrow & & & & \downarrow & \downarrow \\ \theta & \theta & & & & \theta & \theta \end{array}$$

There are FOUR paths of change, and thus, the Multivariable Chain Rule must be the sum of four products.

$$\frac{\partial w}{\partial \theta}=\frac{\partial f}{\partial x}\frac{\partial x}{\partial s}\frac{\partial s}{\partial \theta}+\frac{\partial f}{\partial x}\frac{\partial x}{\partial t}\frac{\partial t}{\partial \theta}+\frac{\partial f}{\partial y}\frac{\partial y}{\partial s}\frac{\partial s}{\partial \theta}+\frac{\partial f}{\partial y}\frac{\partial y}{\partial t}\frac{\partial t}{\partial \theta}.$$

(#3) The Implicit Derivative formula for the 3-variable surface $F(x,y,z)=0$ is

$$\frac{\partial y}{\partial x}=-\left(\frac{F_x}{F_y}\right).$$

If we have the surface $xy^2+xz+x^2\cos(xz)=3+\pi$, then evaluate

$$\frac{\partial z}{\partial y}\ \text{ when } x=1,\ y=2,\ \text{and } z=\pi.$$

By rotating the letters around, we have

$$\frac{\partial z}{\partial y}=-\left(\frac{F_y}{F_z}\right).$$

We move everything to the left side to create the 3-variable parent function.

$$F(x,y,z)=xy^2+xz+x^2\cos(xz)-3-\pi=0.$$

Here are the two partials:

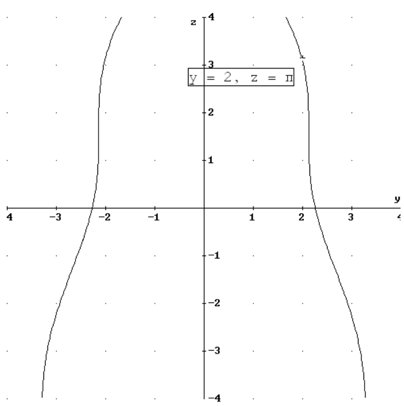
$$\begin{aligned} F_y &= 2xy \\ F_z &= x+x^2\frac{\partial}{\partial z}[\cos(xz)]=x+x^2\left(-\sin(xz)\frac{\partial}{\partial z}[xz]\right)=x-x^3\sin(xz). \end{aligned}$$

The implicit dervative is

$$\frac{\partial z}{\partial y}=-\left(\frac{2xy}{x-x^3\sin(xz)}\right)=-\left(\frac{2(1)(2)}{1-(1^3)\sin(1*\pi)}\right)=-4$$

If we hold x constant at $x=1$, then the trace (slice) would look like this:

$y^2+z+\cos(z)=3+\pi$



Clearly, at the point of interest, the slope of the tangent line is negative. The value $m=-4$ seems reasonable.