

Solutions to Quiz #02 – MATH 2421

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1. Evaluate this cross product (presumably using determinants):

$$\begin{aligned} \langle -1, 3, 2 \rangle \times \langle 4, 1, 5 \rangle &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & 2 \\ 4 & 1 & 5 \end{vmatrix} \\ &= \mathbf{i} \begin{vmatrix} 3 & 2 \\ 1 & 5 \end{vmatrix} - \mathbf{j} \begin{vmatrix} -1 & 2 \\ 4 & 5 \end{vmatrix} + \mathbf{k} \begin{vmatrix} -1 & 3 \\ 4 & 1 \end{vmatrix} \\ &= \mathbf{i}(15 - 2) - \mathbf{j}(-5 - 8) + \mathbf{k}(-1 - 12) = \langle 13, 13, -13 \rangle. \end{aligned}$$

2. We have two points on a line in 3D space. They are $P(2, -2, 5)$ and $Q(3, -2, -4)$.

Give the parametric equations for this line such that when $t = 0$, we are located at Q .

The direction vector is \overrightarrow{PQ} (or \overrightarrow{QP}), $\langle a, b, c \rangle = \langle 1, 0, -9 \rangle$.

The base point is $(x_1, y_1, z_1) = (3, -2, -4)$.

$$\begin{aligned} x &= (1)t + (3) = t + 3 \\ y &= (0)t + (-2) = -2 \\ z &= (-9)t + (-4) = -9t - 4. \end{aligned}$$

3. $\|\boldsymbol{\tau}\| = \|\mathbf{r} \times \mathbf{F}\| = \|\mathbf{r}\| * \|\mathbf{F}\| * \sin(\alpha)$

Suppose we have a long 2-foot wrench which will rotate a bolt. A 30-pound force is applied at the end of the wrench at an angle of 55° with respect to the wrench as shown below.



- (a) What is the angle of separation between \mathbf{r} and \mathbf{F} ? We translate the force vector so that \mathbf{r} and \mathbf{F} are tail-to-tail.

It actually defines a parallelogram, but I didn't sketch in the remaining side. The angles are supplementary, so the true angle of separation is $(180^\circ - 55^\circ) = 125^\circ = \alpha$.

- (b) In which direction does the torque vector point?

The shortest angular path from \mathbf{r} to \mathbf{F} is **CLOCKWISE** (Righty-Tighty), so by the Right Hand Rule, the torque vector must point **INTO THE PAPER**.

(c) Give a simplified algebraic expression for the MAGNITUDE of the torque, $\|\boldsymbol{\tau}\|$.

$$(2 \text{ ft})(30 \text{ lb}) \sin(125^\circ) = 60 \sin(125^\circ) \text{ ft-lb.}$$

If you said $60 \sin(55^\circ)$ ft-lb, this is okay, but you may have answered part (a) incorrectly...

4. $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

(a) Find the standard form for the plane in 3D which is normal to the vector $\mathbf{v} = \langle 0, 1, 0 \rangle$ and contains the point $P(2, 3, 4)$.

We have $a = 0$, $b = 1$, and $c = 0$.

$$\begin{aligned} 0(x - 2) + 1(y - 3) + 0(z - 4) &= 0 \\ (y - 3) &= 0. \end{aligned}$$

This is the plane $y = 3$.

(b) Multiple choice. Circle the correct answer.

The plane found in part (a) is PARALLEL to which of these planes?

That plane is parallel to the **(ii)** xz -plane. The equation of the xz -plane is $y = 0$.