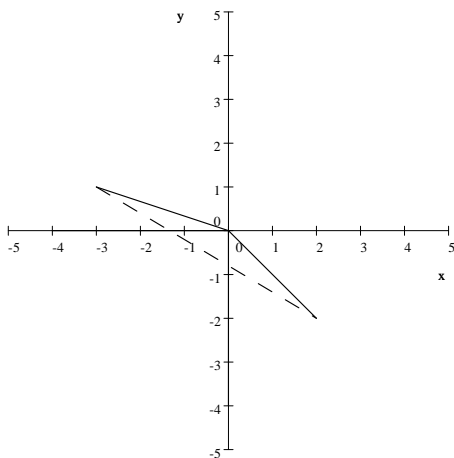


Solutions to Quiz #01 – MATH 2421

Puhalskii

1. Sketch the following 2D vector with THEIR TAILS AT THE ORIGIN.

$$\mathbf{a} = \langle -3, 1 \rangle \text{ and } \mathbf{b} = \langle 2, -2 \rangle.$$



Now find $(\mathbf{a} - \mathbf{b})$ algebraically and give its component form here.

$$\mathbf{a} - \mathbf{b} = \langle -3 - 2, 1 - (-2) \rangle = \langle -5, 3 \rangle.$$

Now sketch in $(\mathbf{a} - \mathbf{b})$ in the diagram by connecting the appropriate points. The initial point is the head of \mathbf{b} and the terminal point is the head of \mathbf{a} .

2. Let $\mathbf{a} = \langle 2, -1, 3 \rangle$.

- (a) Find the unit vector associated with \mathbf{a} which has the same direction as \mathbf{a} .

$$\mathbf{u} = \frac{\mathbf{a}}{\|\mathbf{a}\|} = \frac{1}{\sqrt{2^2 + (-1)^2 + 3^2}} \langle 2, -1, 3 \rangle = \frac{1}{\sqrt{14}} \langle 2, -1, 3 \rangle.$$

- (b) Now find the vector which has magnitude 5 and has the OPPOSITE direction as \mathbf{a} . Multiply by the scalar (-5) .

$$-\frac{5}{\sqrt{14}} \langle 2, -1, 3 \rangle.$$

3. Here is the equation for a 3D sphere (complete the square!):

$$\begin{aligned} x^2 + y^2 - 10y + z^2 &= 24 \\ x^2 + y^2 - 10y + (-5)^2 + z^2 &= 24 + (-5)^2 \\ (x - 0)^2 + (y - 5)^2 + (z - 0)^2 &= 49 = 7^2 \end{aligned}$$

- (a) Where is its center located? $C(0, 5, 0)$.

- (b) What is the length of the radius? $R = 7$.

4. Use the geometric form of the dot product to find the angle of separation between

$$\mathbf{a} = \langle 1, 1 \rangle \text{ and } \mathbf{b} = \langle 4, 3 \rangle. \text{ We have}$$

$$\alpha = \cos^{-1} \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} \right) = \cos^{-1} \left(\frac{(1)(4) + (1)(3)}{\sqrt{1^2 + 1^2} \sqrt{4^2 + 3^2}} \right) = \cos^{-1} \left(\frac{7}{5\sqrt{2}} \right) \doteq 8.1^\circ.$$