Some Solutions to Assignment #05 – MATH 2421
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Section 10.4

(#24) A gun is fired with angle of elevation 30°. What is the muzzle speed if the maximum height of the shell is 500 m.

We need the parametric equations. The unknown is v_0, the muzzle speed.

$$\begin{aligned} x(t) &= v_0 \cos(30^\circ)\, t \\ y(t) &= -4.9t^2 + v_0 \sin(30^\circ)\, t \end{aligned}$$

When the shell is at maximum height, we must have $y'(t) = 0$.

$$y'(t) = -9.8t + v_0 \sin(30^\circ) = 0$$

This occurs at when? Solve for t.

$$v_0 \sin(30^\circ) = 9.8t \Rightarrow t = \frac{v_0 \sin(30^\circ)}{9.8}.$$

We substitute this back into the original $y(t)$ equation.

$$\begin{aligned} -4.9\left(\frac{v_0 \sin(30^\circ)}{9.8}\right)^2 + v_0 \sin(30^\circ)\left(\frac{v_0 \sin(30^\circ)}{9.8}\right) &= 500 \\ -\left(\frac{4.9}{9.8^2}\right)\left(\frac{1}{4}\right) v_0^2 + \left(\frac{1}{9.8}\right)\left(\frac{1}{4}\right) v_0^2 &= 500 \\ \left(-\frac{1}{2}\right) v_0^2 + v_0^2 &= 500\,(4 * 9.8) \\ \frac{1}{2} v_0^2 &= 500\,(4 * 9.8) \\ v_0^2 &= 4000 * 9.8 \end{aligned}$$

Thus, we have $v_0 = \sqrt{4000 * 9.8} = 198$ m/sec.

Section 11.3

(#14) $\frac{\partial}{\partial x}\left[x^5 + 3x^3y^2 + 3xy^4\right] = \frac{\partial}{\partial x}\left[x^5\right] + 3y^2 \frac{\partial}{\partial x}\left[x^3\right] + 3y^4 \frac{\partial}{\partial x}[x] = 5x^4 + 3y^2\left(3x^2\right) + 3y^4(1) =$

$5x^4 + 9x^2y^2 + 3y^4$.

$\frac{\partial}{\partial y}\left[x^5 + 3x^3y^2 + 3xy^4\right] = \frac{\partial}{\partial y}\left[x^5\right] + 3x^3 \frac{\partial}{\partial y}\left[y^2\right] + 3x \frac{\partial}{\partial y}\left[y^4\right] = 0 + 3x^3(2y) + 3x\left(4y^3\right) =$

$6x^3y + 12xy^3$.

(#16) $\frac{\partial}{\partial x}\left[y\ln(x)\right] = y\frac{\partial}{\partial x}\left[\ln(x)\right] = y\left(\frac{1}{x}\right) = \frac{y}{x}.$

$$\frac{\partial}{\partial y}\left[y\ln(x)\right] = \ln(x)\frac{\partial}{\partial y}\left[y\right] = \ln(x) * 1 = \ln(x).$$

(#18) $\frac{\partial}{\partial x}\left[x^{y}\right] = yx^{y-1}.$

$$\frac{\partial}{\partial y}\left[x^{y}\right] = x^{y}\left(\ln(x)\right).$$

(#20) Quotient Rule.

$$\frac{\partial}{\partial s}\left[\frac{st^2}{s^2+t^2}\right] = t^2 * \frac{\partial}{\partial s}\left[\frac{s}{s^2+t^2}\right] = t^2\left(\frac{\left(s^2+t^2\right)\frac{\partial}{\partial s}\left[s\right] - s * \frac{\partial}{\partial s}\left[s^2+t^2\right]}{\left(s^2+t^2\right)^2}\right) =$$

$$t^2\left(\frac{\left(s^2+t^2\right)(1) - s\left(2s\right)}{\left(s^2+t^2\right)^2}\right) = \frac{t^2\left(t^2-s^2\right)}{\left(s^2+t^2\right)^2}.$$

$$\frac{\partial}{\partial t}\left[\frac{st^2}{s^2+t^2}\right] = s * \frac{\partial}{\partial t}\left[\frac{t^2}{s^2+t^2}\right] = s\left(\frac{\left(s^2+t^2\right)\frac{\partial}{\partial t}\left[t^2\right] - t^2 * \frac{\partial}{\partial t}\left[s^2+t^2\right]}{\left(s^2+t^2\right)^2}\right) =$$

$$s\left(\frac{\left(s^2+t^2\right)(2t) - t^2\left(2t\right)}{\left(s^2+t^2\right)^2}\right) = \frac{2s^3t}{\left(s^2+t^2\right)^2}.$$

(#22) $\frac{\partial}{\partial x}\left[e^{\sin(t/x)}\right] = e^{\sin(t/x)}\frac{\partial}{\partial x}\left[\sin\left(\frac{t}{x}\right)\right] = e^{\sin(t/x)}\cos\left(\frac{t}{x}\right) * \frac{\partial}{\partial x}\left[\frac{t}{x}\right] =$

$$e^{\sin(t/x)}\cos\left(\frac{t}{x}\right)\left(-\frac{t}{x^2}\right) = -\left(\frac{t}{x^2}\right)e^{\sin(t/x)}\cos\left(\frac{t}{x}\right).$$

$$\frac{\partial}{\partial t}\left[e^{\sin(t/x)}\right] = e^{\sin(t/x)}\frac{\partial}{\partial t}\left[\sin\left(\frac{t}{x}\right)\right] = e^{\sin(t/x)}\cos\left(\frac{t}{x}\right) * \frac{\partial}{\partial t}\left[\frac{t}{x}\right] =$$

$$\left(\frac{1}{x}\right)e^{\sin(t/x)}\cos\left(\frac{t}{x}\right).$$

(#30) $\frac{\partial}{\partial x}\left[x^{y/z}\right] = \left(\frac{y}{z}\right)x^{(y/z)-1}$

$$\frac{\partial}{\partial y}\left[x^{y/z}\right] = x^{y/z}\ln(x) * \frac{\partial}{\partial y}\left[\frac{y}{z}\right] = \left(\frac{1}{z}\right)x^{y/z}\ln(x).$$

$$\frac{\partial}{\partial z}\left[x^{y/z}\right] = x^{y/z}\ln(x) * \frac{\partial}{\partial z}\left[\frac{y}{z}\right] = -\left(\frac{y}{z^2}\right)x^{y/z}\ln(x).$$

(#44) $F(x,y,z) = xy^2z^3 + x^3y^2z - x - y - z = 0.$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{y^2z^3 + 3x^2y^2z - 1}{3xy^2z^2 + x^3y^2 - 1}.$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{2xyz^3 + 2x^3yz - 1}{3xy^2z^2 + x^3y^2 - 1}.$$

(#48) Find all three second partial derivatives.

$$f_x = \frac{\partial}{\partial x}\left[\ln(3x+5y)\right] = \frac{\frac{\partial}{\partial x}[3x+5y]}{3x+5y} = \frac{3}{3x+5y}.$$

$$\begin{aligned} f_{xx} &= \frac{\partial}{\partial x}[f_x] = \frac{\partial}{\partial x}\left[\frac{3}{3x+5y}\right] = 3\frac{\partial}{\partial x}\left[(3x+5y)^{-1}\right] \\ &= 3\left(-1(3x+5y)^{-2}\frac{\partial}{\partial x}[3x+5y]\right) = -\frac{9}{(3x+5y)^2}. \end{aligned}$$

$$\begin{aligned} f_{xy} &= \frac{\partial}{\partial y}[f_x] = \frac{\partial}{\partial y}\left[\frac{3}{3x+5y}\right] = 3\frac{\partial}{\partial y}\left[(3x+5y)^{-1}\right] \\ &= 3\left(-1(3x+5y)^{-2}\frac{\partial}{\partial y}[3x+5y]\right) = -\frac{15}{(3x+5y)^2}. \end{aligned}$$

$$f_y = \frac{5}{3x+5y}.$$

$$\begin{aligned} f_{yy} &= \frac{\partial}{\partial y}[f_y] = \frac{\partial}{\partial y}\left[\frac{5}{3x+5y}\right] = 5\frac{\partial}{\partial y}\left[(3x+5y)^{-1}\right] \\ &= 5\left(-1(3x+5y)^{-2}\frac{\partial}{\partial y}[3x+5y]\right) = -\frac{25}{(3x+5y)^2}. \end{aligned}$$