Practice Material for Second Midterm for Calculus III – MATH 2421
Fall 2005

Kawai/Gould

(#1) Give the limits of integration for ALL points in Octant I:

(a) in Rectangular Coordinates, $dz\ dy\ dx$.
(b) in Cylindrical Coordinates, $dz\ r\ dr\ d\theta$.
(c) in Spherical Coordinates, $\rho^2 \sin(\phi)\ \ d\rho\ d\phi\ d\theta$.

(#2) If C is the line segment which starts at $(2, 3, -5)$ and ends at $(2, 3, 4)$, then which of these quantities MUST equal $\int_C Pdx + Qdy + Rdz$?

(a) zero
(b) $\int_C Pdx$
(c) $\int_C Qdy$
(d) $\int_C Rdz$
(e) $\iint_R (Q_x - P_y)\, dA$, where R is the interior of C.

(#3) Consider these carefully...

(a) TRUE OR FALSE? Green's Theorem will work when the field is $\mathbf{F} = \left\langle \frac{1}{y}, \frac{1}{x} \right\rangle$ and C is $x^2 + y^2 = 1$.
(b) If every simple path C from point A to point B (A and B are arbitrary) gives us the same answer for

$$\int_C Pdx + Qdy,$$

then is $\mathbf{F}(x, y) = \langle P(x, y), Q(x, y) \rangle$ necessarily a conservative field over the domain where the C's are defined?
(c) If at least one simple closed path C from P to Q gives us

$$\oint_C Pdx + Qdy = 0,$$

then is $\mathbf{F}(x, y) = \langle P(x, y), Q(x, y) \rangle$ necessarily a conservative field over the domain where the C's are defined?

(#4) Suppose $\mathbf{F}(x,y,z) = \left\langle \frac{y}{z} + z\sin(x),\ \frac{x}{z},\ -\frac{xy}{z^2} - \cos(x) \right\rangle$.

(a) We already know that $\mathbf{F}$ is conservative. Find a potential function $f(x,y,z)$.

(b) It turns out for this vector field, there IS a slight restriction concerning where the path C may traverse.

Complete the following warning:

The value of $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path as long as C does not intersect the _____________?

(c) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along any path which abides by part (b), and starts at $(0,4,1)$ and ends at $\left(\frac{\pi}{6}, 6, 2\right)$.

(#5) Find the mass of half of a canteloupe:

The solid region E lies between the spheres $x^2+y^2+z^2 \le 1$ and $x^2+y^2+z^2 \le 9$ and we have $z \ge 0$.

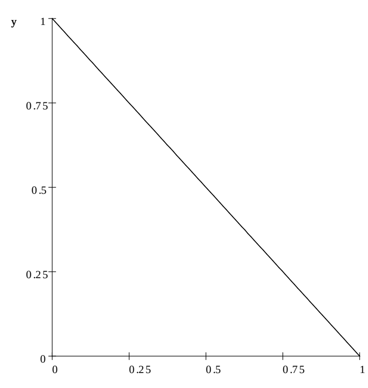
The density function is $\sigma(x,y,z) = \frac{z}{\sqrt{x^2+y^2+z^2}}$. [This is actually $\cos(\phi)$.]

(#6) Suppose that we tell you that $\mathbf{F}(x,y,z) = \langle y\sin(z) + x^2,\ x\sin(z),\ xy\cos(z)\rangle$ is conservative.

(a) Find a potential function $f(x,y,z)$ such that $\nabla f = \mathbf{F}$.

(b) Find the work accomplished by $\mathbf{F}$ on a particle moving from $(0,0,0)$ to $(1,1,0)$ along the path $\mathbf{r}(t) = \langle t, t^2, 0\rangle$.

(#7) Find the center of mass of the planar laminum triangle:



The density function is $\sigma(x,y) = xy^2$.

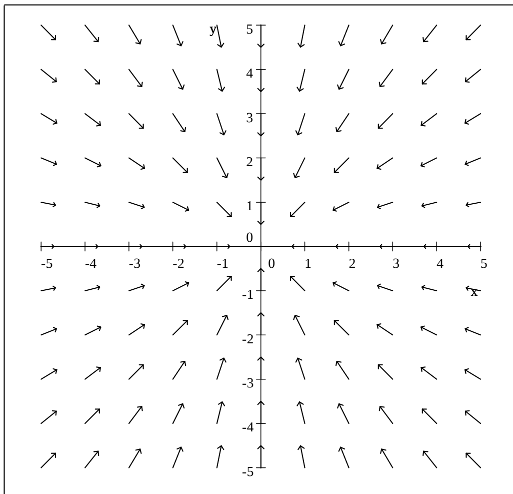
Thus, you need to find m, $\overline{x}$, and $\overline{y}$.

(#8) Remember that the diagrams are scaled characterizations. The directions are correct, and relative magnitude for the vectors are correct. The absolute lengths are scaled.

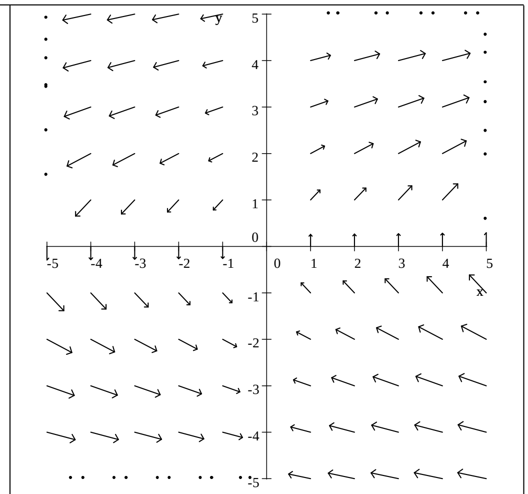
(a) Which one of the vector fields most closely resembles $\mathbf{F}(x,y) = \left\langle -\dfrac{x}{\sqrt{x^2+y^2}}, -\dfrac{y}{\sqrt{x^2+y^2}} \right\rangle$?

(b) Which one of the vector fields most closely resembles $\mathbf{F}(x,y) = \langle x+y,\ x-y \rangle$?

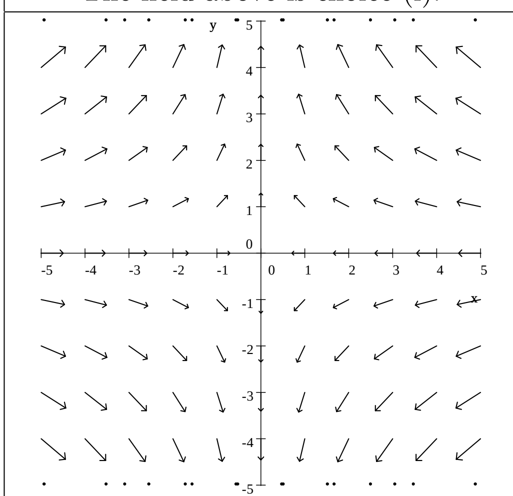
(c) Which one of the vector fields most closely resembles $\mathbf{F}(x,y) = \langle xy,\ x \rangle$?



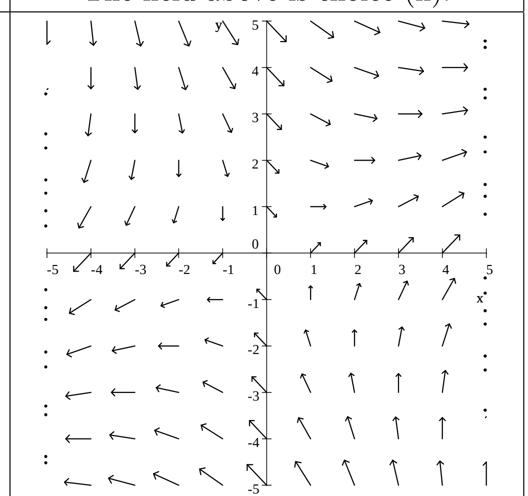
The field above is choice (i).



The field above is choice (ii).



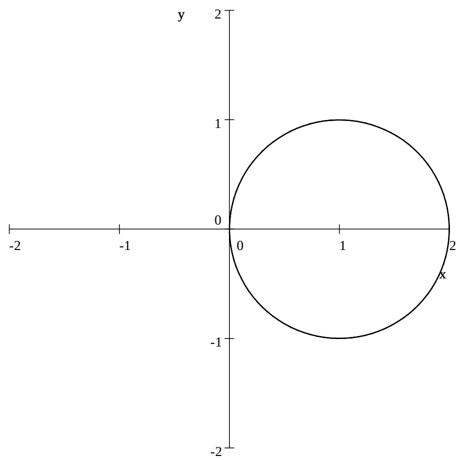
The field above is choice (iii).



The field above is choice (iv).

(#9) Different integrals.

(a) Find the surface area of the portion of the cone $z = \sqrt{x^2 + y^2}$ above the circle $(x-1)^2 + y^2 = 1$.



This is easy once you have the surface area differential.

[Hint: This is a double integral.]

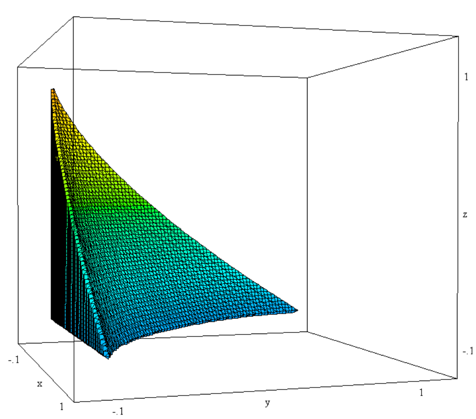
(b) Let E be the volume trapped beneath that cone and above that circle.
Find the mass of E if the density function is $\sigma(x, y, z) = 2z$.
[Hint: This one must be a triple integral.]

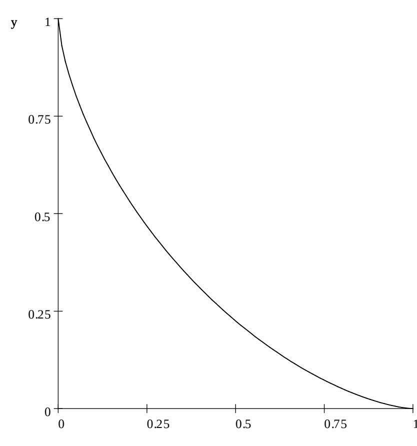
(#10) This is a very intricate u-substitution. We probably wouldn't give you something this complicated, but it's a good practice problem.

Consider the surface $z = 1 - x^{2/3} - y^{2/3}$. We can trap the volume beneath it in Octant I.

Notice that the intersection of this surface with the xy-plane IS the Astroid,

$$x^{2/3} + y^{2/3} = 1 \Rightarrow y = \left(1 - x^{2/3}\right)^{???}.$$





(a) Write the DOUBLE integral which describes the trapped volume using the order of integration $dy\,dx$.

(b) Evaluate the INNER integral carefully. Watch your exponents! Remember that the answer to this step should contain x's.

(#11) Find the work performed by the force field $\mathbf{F}(x,y) = \left\langle y^2,\ e^x \right\rangle$.

The path is defined by C: $y = x^2$ from $(0,0)$ to $(2,4)$.

Hint: You should need to do integration-by-parts once.

(#12) Find the mass of this wire.

C : $x^2 + y^2 = 4$, in Quadrant I only.

The density function is $\sigma(x,y) = 2x$.

$$m = \int_C 2x\ ds.$$

(#13) For each of the two-variable functions below, determine the NUMBER of critical points in $\mathbb{R}^2$.

(a) $z = f(x,y) = 2 - \sqrt{x^2 + y^2}$

(b) $z = f(x,y) = (x+y)\,e^{-x^2-y^2}$. Be careful!

(#14) Let $f(x,y) = \left(x^2 + y\right) e^{y/2}$. There is only ONE critical point. Find it and classify it according to the Second Partials Test.

(#15) Due to the current order of integration, the following double integral is impossible to evaluate:

$$\int_0^1 \int_0^{\cos^{-1}(y)} \sqrt{\sin(x)}\ dx\ dy.$$

Use Fubini's Theorem to evaluate this double integral.