Final Exam for Analytic Geometry and Calculus III – MATH 2421

Fall 2004

Name: ______________________________ ID#: ______________________________

Circle the name of your instructor: Mike Kawai / Jason Gould.

Directions:

1. No calculators, computers, books, or external notes. You may use the notesheet provided.
2. Box/circle/highlight your final answers.
3. You may use the back of the sheets as scratch paper, but please indicate clearly where your work is located for each problem.
4. You have three hours to complete the exam. Enjoy.

	Your Score	Possible Points		Your Score	Possible Points
Page 2		15	Page 6		18
Page 3		10	Page 7		14
Page 4		15	Page 8		14
Page 5		16			
			TOTAL		102

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(#1) The answer to each of the clues is either:

(a) the number zero, 0.

(b) the zero vector, $\mathbf{0}$.

(c) not necessarily (a) or (b).

[2 *pts. each*] So write (a), (b), or (c) in each of the corresponding blanks.

(i) When $\mathbf{u}$ and $\mathbf{v}$ are perpendicular, the value of $\mathbf{u} \cdot \mathbf{v}$ is ___.

(ii) If $\mathbf{F}(x, y, z)$ is a conservative vector field, then the value of $\nabla \cdot \mathbf{F}$ is ___.

(iii) If $\mathbf{u} = \mathbf{v} \times \mathbf{w}$, then the value of $\mathbf{Proj_v u}$ is ___.

(#2) Work, work, work...

An object is moved from $P(1, 3, -5)$ to $Q(4, 4, -4)$ along a straight line segment.

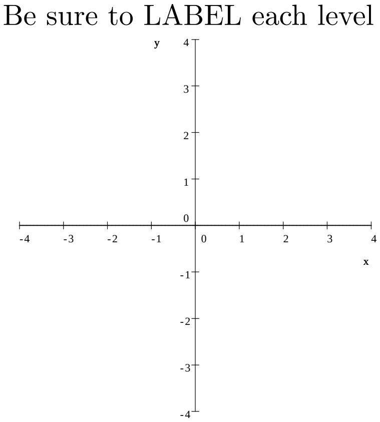
(a) [3 *pts.*] Give the parameterization of the path (from P to Q) using $0 \leq t \leq 1$.
In other words, $\mathbf{r}(t) =$???

(b) [2 *pts.*] If the force vector is constant, $\mathbf{F} = \langle 3, 6, 5 \rangle$, then how much work does $\mathbf{F}$ accomplish moving the object from P to Q?

(c) [4 *pts.*] Suppose $\mathbf{F}$ is the force field $\mathbf{F}(x, y, z) = \langle x^2, \ -y, \ z \rangle$.
Write down (BUT DO NOT EVALUATE) the appropriate integral in t which describes the amount of work which $\mathbf{F}$ accomplishes moving the object from P to Q.

(#3) Suppose we have a 2-variable function $f(x, y) = y - x^2$.

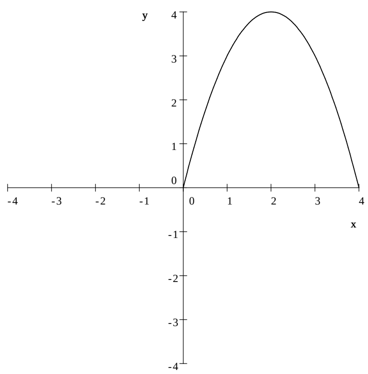
(a) [2 *pts.*] On the axes below, sketch in the level curves $f(x, y) = k$ for $k = 0$ and $k = 2$.
Be sure to LABEL each level curve with its k-value.



(b) [2 *pts.*] It should be clear to you that the point $(1, 1)$ must lie on the level curve associated with $k = 0$, since $f(1, 1) = 0$. Sketch in $\nabla f(1, 1)$ at the point $(1, 1)$. Be sure that it is drawn to scale (has the correct length).

(c) [2 *pts.*] Suppose we wanted to find the relative extrema for f.
Show that f has NO critical points.

(d) [2 *pts.*] Now suppose that our domain of interest (D) is the area $0 \leq y \leq -x^2 + 4x$.



Parameterize the curvy edge of D and determine if there are any extreme values of f along this edge, but not at the corners.
Hint: $(0,0)$ does NOT count since it is a corner point!

(e) [2 *pts.*] If you didn't find anything in part (d), then you don't get anything for this part. Sorry.
Assuming that you did, was the interesting point along that boundary a maximum or a minimum?

Explain why this point is an extremum for the entire region D. Hint: Results from part (a)?

Multiple choice questions are worth 3 points each!

(#4) Suppose f is a function of u, v, and w.
Suppose $u(r,\theta) = r\theta$ and $v(r) = e^r$ and $w(r,\theta) = \frac{\theta}{r}$.
Which expression below is equivalent to $\dfrac{\partial f}{\partial \theta}$?

(a) $\dfrac{\partial}{\partial u}\left[\dfrac{\partial u}{\partial \theta}\right] + \dfrac{\partial}{\partial v}\left[\dfrac{\partial v}{\partial \theta}\right] + \dfrac{\partial}{\partial w}\left[\dfrac{\partial w}{\partial \theta}\right]$

(b) $\dfrac{\partial f}{\partial u}\dfrac{\partial u}{\partial \theta} + \dfrac{\partial f}{\partial v}\dfrac{\partial v}{\partial \theta} + \dfrac{1}{r}\left(\dfrac{\partial f}{\partial w}\dfrac{\partial w}{\partial \theta}\right)$

(c) $\dfrac{\partial f}{\partial u}(r) + \dfrac{\partial f}{\partial w}\left(\dfrac{1}{r}\right)$

(d) $\left(\dfrac{\partial f}{\partial u}\dfrac{\partial u}{\partial \theta}\right)\left(\dfrac{\partial f}{\partial v}\dfrac{\partial v}{\partial \theta}\right)\left(\dfrac{\partial f}{\partial w}\dfrac{\partial w}{\partial \theta}\right)$

(e) None of these.

(#5) We already know that the function $f(x,y) = 3x^2 + 3xy + y^2$ has a critical point at $(0,0)$.
What is the classification of this critical point?

(a) relative minimum
(b) relative maximum
(c) saddle point
(d) It cannot be determined from the information given.

(#6) If $\mathbf{F} = \left\langle x - z,\ xz^2,\ \dfrac{y}{z} \right\rangle$, what is the value of $\nabla \times \mathbf{F}$?

(a) $-2xz + \dfrac{1}{z} + 1 + z^2$

(b) $\left\langle -2xz + \dfrac{1}{z},\ -1,\ z^2 \right\rangle$

(c) $\left\langle -2xz + \dfrac{1}{z},\ 1,\ z^2 \right\rangle$

(d) $\langle 1,\ 0,\ 0 \rangle$

(e) None of these.

(#7) If $(x, y, z) = \left(-4,\ 4,\ -4\sqrt{2}\right)$, then $(\rho, \theta, \phi) =$???

(a) $\left(8, \dfrac{\pi}{4}, \dfrac{\pi}{4}\right)$

(b) $\left(8, \dfrac{\pi}{4}, \dfrac{3\pi}{4}\right)$

(c) $\left(8, \dfrac{3\pi}{4}, \dfrac{\pi}{4}\right)$

(d) $\left(8, \dfrac{3\pi}{4}, \dfrac{3\pi}{4}\right)$

(e) None of these.

(#8) If the unit tangent vector is $\mathbf{T}(t) = \left\langle \dfrac{1}{\sqrt{1+t^2}}, \dfrac{t}{\sqrt{1+t^2}} \right\rangle$, then the unit normal vector $\mathbf{N}(t) =$???

(a) $\left\langle \dfrac{-1}{\sqrt{1+t^2}}, \dfrac{t}{\sqrt{1+t^2}} \right\rangle$

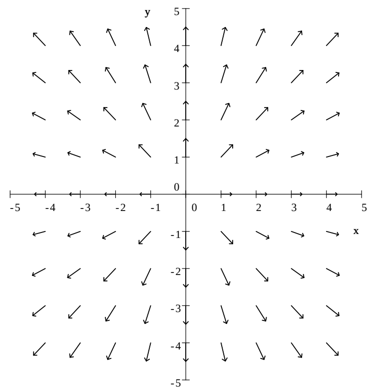
(b) $\left\langle \dfrac{1}{\sqrt{1+t^2}}, \dfrac{t}{\sqrt{1+t^2}} \right\rangle$

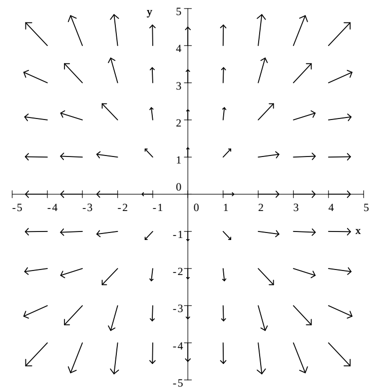
(c) $\left\langle \dfrac{-t}{\sqrt{1+t^2}}, \dfrac{1}{\sqrt{1+t^2}} \right\rangle$

(d) $\left\langle \dfrac{t}{\sqrt{1+t^2}}, \dfrac{-1}{\sqrt{1+t^2}} \right\rangle$

(#9) [3 *pts.*] Examine the two vector fields below. One of them is a good representation of $\mathbf{F}(x,y) = \left\langle \frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}} \right\rangle$. The other one is unreasonable, considering the magnitudes displayed.

Circle the one which best represents $\mathbf{F}$ and give a one-sentence justification for your choice.





(#10) [6 *pts.*] Suppose we have the surface $x\sin(z) - \cos(yz) = 2 - \frac{\sqrt{3}}{2}$.

Find $\frac{\partial z}{\partial x}$ evaluated at $\left(2, \frac{1}{3}, \frac{\pi}{2}\right)$.

(#11) Let $\mathbf{F}(x,y) = \left\langle xe^x + 2xy,\ x^2 + y^3 \right\rangle$.

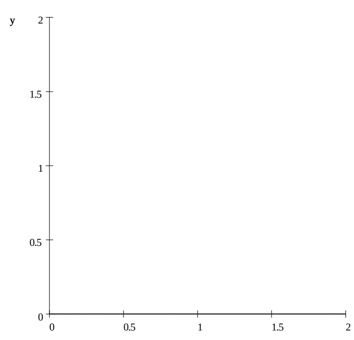
(a) [3 *pts.*] Show that $\mathbf{F}$ is conservative.

(b) [4 *pts.*] Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ for any simple path which begins at $P(0,0)$ and ends at $Q(\ln(2), 1)$.

(#12) As is stands, the following double integral is impossible to evaluate...

$$\int_0^1 \int_y^1 e^{x^2}\,dx\,dy = ???$$

(a) [2 *pts.*] First, sketch the region of integration R.



(b) [5 *pts.*] Now use Fubini to evaluate the double integral.

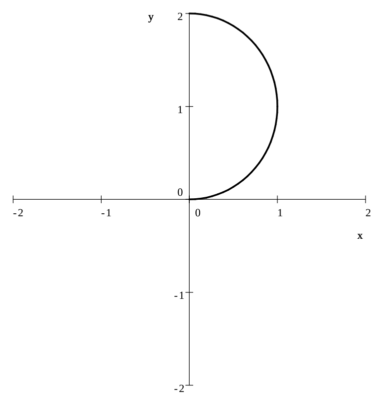
(#13) [7 *pts.*] If $f(x,y,z) = xy + xz + yz$, find the value of the total differential df if x moves from 2 to 2.2, y moves from 3 to 2.9, and z moves from 4 to 4.01.

(#14) [4 *pts.*] Find the (rectangular coordinates) equation of the plane which is perpendicular to the yz-plane and contains the origin and the point $(0, 1, 2)$.

(#15) [7 *pts.*] Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ if $\mathbf{F} = \left\langle \sin\left(x^3\right), \frac{x^3}{3} + xy^2 + \ln(1+y) \right\rangle$ and C is the closed path which bounds the semicircle $x^2 + y^2 \leq 1$ with $y \geq 0$.

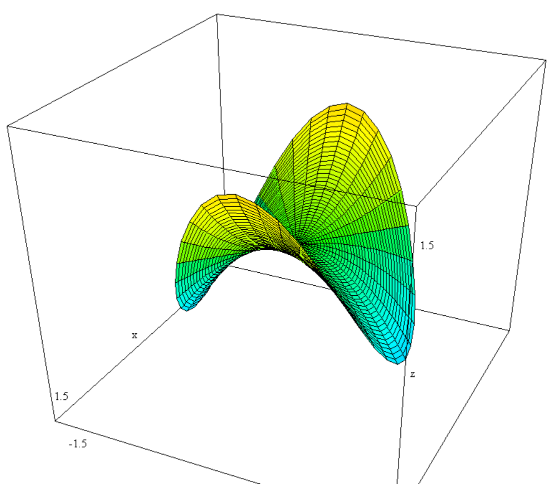
(#16) [7 *pts.*] The region of integration R is the portion of the circle in Quadrant I whose center is located at $(0, 1)$ with radius 1.

If $z = g(x, y) = \sqrt{x^2 + y^2}$ (the upper cone), and the density function is $\sigma(x, y, z) = x$. Find the mass for the portion of the cone which is above R.



(#17) [7 *pts.*] Let S be the portion of the hyperbolic paraboloid $z = x^2 - y^2$ inside the right circular cylinder $x^2 + y^2 = 1$.

Thus, S is somewhat shaped like a Pringles potato crisp. Yum.



Let $\mathbf{F}(x, y, z) = \left\langle \frac{1}{2}x,\ 3y,\ z \right\rangle$ represent the radiation field when the Pringles are "sterilized". Yum. Let $\mathbf{n}$ be the *upward* normal with respect to S. Calculate the upward radiation flux passing through S.

$\sin^2(\theta) = \frac{1}{2} - \frac{\cos(2\theta)}{2}$.

(#18) [7 *pts.*] Let Q be the solid hemisphere $x^2 + y^2 + z^2 \leq 1$ with $z \geq 0$, and let S be its surface.

Let $\mathbf{F}(x, y, z) = \left\langle xze^z,\ -yze^z,\ z^2 \right\rangle$ and $\mathbf{n} =$ the outward normal with respect to S.

Evaluate $\oint_S (\mathbf{F} \cdot \mathbf{n})\, dS$.