MATH 2421 (Test on Ch. 9 & 10)
Spring 2004 – half.tex

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Instructions. Show all work on free response questions. The numbers used in the calculations are relatively small. No decimal approximations to square roots are necessary. Circle the *best* answer for multiple-choice questions (no need to show work).

The multiple choice questions are worth 5 pts. each.

(#1) In cylindrical coordinates, the *surface* $r = \cos(\theta)$, for $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, assumes that z can be anything. How should we name this surface?

(a) ellipsoid
(b) sphere
(c) right circular cylinder
(d) right cardiodic cylinder
(e) plane.

(#2) With which angle θ do we associate the plane $y = -x$?

(a) $\theta = 0$
(b) $\theta = \frac{\pi}{4}$
(c) $\theta = \frac{\pi}{2}$
(d) $\theta = \frac{3\pi}{4}$
(e) $\theta = \pi$

(#3) If $\mathbf{u} = \langle -2, 0, -4 \rangle$ and $\mathbf{v} = \langle 1, -1, 1 \rangle$, then which vector is equal to $\mathbf{Proj_v u}$?

(a) $\langle 1, -1, 1 \rangle$
(b) $\langle -2, 2, -2 \rangle$
(c) $\langle 2, -2, 2 \rangle$
(d) $\langle -6, 6, -6 \rangle$
(e) $\langle 6, -6, 6 \rangle$

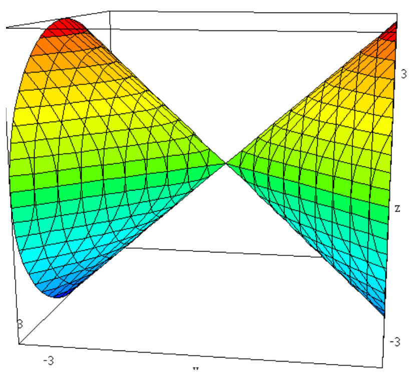
(#4) A cable runs from $P(1, 0)$ to $Q(6, 12)$. The tension vector has the same direction as $\overrightarrow{PQ}$, but has magnitude 100 pounds. Which vector below represents the tension vector?

(a) $\frac{100}{13} \langle 5, 12 \rangle$
(b) $\frac{100}{\sqrt{13}} \langle 5, 12 \rangle$
(c) $\frac{13}{100} \langle 5, 12 \rangle$
(d) $\left\langle \frac{500}{169}, \frac{1200}{169} \right\rangle$
(e) $100 \left\langle \frac{6}{\sqrt{180}}, \frac{12}{\sqrt{180}} \right\rangle$

(#5) $x^2 + y^2 + z^2 + 2x - 2y = 25$. What is the radius of this sphere?

(a) 5
(b) 25
(c) $\sqrt{23}$
(d) $3\sqrt{3}$
(e) 27

(#6) Orientation. Which equation best fits the figure?



(a) $z^2 = x^2 + y^2$

(b) $z^2 = x^2 - y^2$

(c) $x^2 = y^2 + z^2$

(d) $y = x^2 + z^2$

(e) $y^2 = x^2 + z^2$

(#7) Suppose we want to move an object from $P\,(1, -3, 0)$ to $Q\,(15, 5, -6)$.

(a) [2 *pts.*] Find $\overrightarrow{PQ}$.

(b) [4 *pts.*] Write down the parametric equations which define the line in 3-D which passes through P and Q.

$x =$

$y =$

$z =$

(c) [4 *pts.*] Suppose that the force $\mathbf{F} = \langle 2, 1, -3 \rangle$ moves the object from P to Q. Find the number of work units accomplished by the force.

(#8) [9 *pts.*] We have two planes: $x + 2y + 3z = 6$ and $-3x + 2y + 6z = 12$.

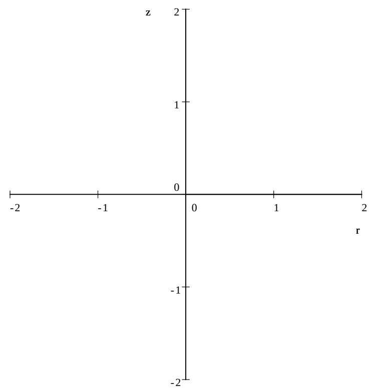
Find the acute $\left(0 \le \alpha < \frac{\pi}{2}\right)$ angle between the planes, also called the dihedral angle.

The answer is *not* one of the standard angles. Just leave your answer in the form of an inverse trig. function...

(#9) The upper cone $z = \sqrt{3}\sqrt{x^2 + y^2}$ can easily be transformed into cylindrical and spherical coordinates.

(a) [2 *pts.*] Make one substitution and find the equation in cylindrical coordinates.

(b) [3 *pts.*] Based on your previous work, show me what the trace (slice) looks like if I choose any of the θ-planes (rz-planes) through the z-axis.

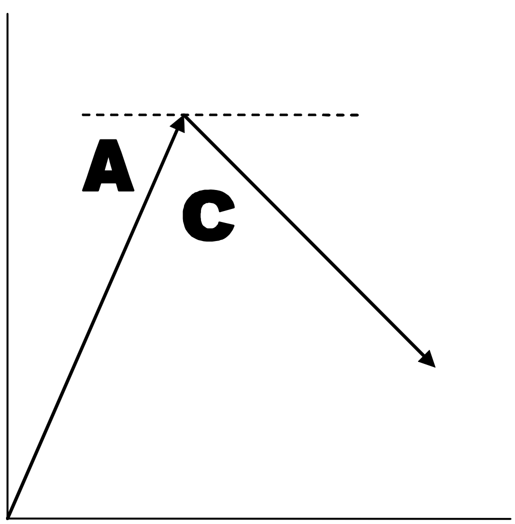


(c) [4 *pts.*] Tougher. By making a few more substitutions and then some cancelling, show that the equation in spherical coordinates is $\phi = a\ constant\ angle$.

(#10) [9 *pts.*] Find the standard form for the plane which contains the points $P\,(0,0,0)$, $Q\,(1,2,-3)$, and $R\,(-2,5,-4)$.

(#11) Here is a picture of the lever arm vector and a force. The level arm vector has its tail positioned at the origin. The force is applied to the head of the level arm vector. We have $|\mathbf{r}| = 20$ feet and $|\mathbf{F}| = 16$ pounds.

Angle A measures 61° and Angle C measures 58°.



(a) [3 *pts.*] Find the angle of separation α, in degrees.

(b) [3 *pts.*] In which direction will the torque vector $\mathbf{T}$ point?

(c) [3 *pts.*] Calculate $|\mathbf{T}|$.

(#12) [2 *pts. each*] True or False? No work is necessary.

(a) ___ If c is a nonzero scalar, then $|c\mathbf{v}|$ always equals $c\,|\mathbf{v}|$.

(b) ___ If $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors and $\mathbf{u}\cdot\mathbf{v}=0$, then $\mathbf{u}$ and $\mathbf{v}$ must be parallel.

(c) ___ If we have the coordinates $(\rho,\theta,\phi)=\left(2,\frac{\pi}{3},\frac{\pi}{2}\right)$, then $z=0$.

(#13) [3 *pts. each*] Match the equations with the appropriate shapes below.

(i) ___ $\rho=7$

(ii) ___ $z=-r^2$

(iii) ___ $x^2-y^2+z^2=4$

(a) sphere

(b) elliptic paraboloid

(c) hyperbolic paraboloid

(d) hyperbolic cylinder

(e) hyperboloid of one sheet

(f) hyperboloid of two sheets

(g) plane

(#14) Suppose we have the position function $\mathbf{r}(t)=\left\langle t,\ 4-t^2\right\rangle$ for $-2\le t\le 2$.

(a) [2 *pt.*] Sketch the 2-D parabola which is associated with this position function.

(b) [2 *pts.*] Sketch in the position vector $\mathbf{r}(1)$ with its tail at the origin.

(c) [3 *pts.*] Calculate $\mathbf{v}(1)$.

(d) [2 *pts.*] Sketch in $\mathbf{v}(1)$ so that we can see the geometric relationship between the velocity vector and the path of the object.