Class Log for MATH 1401-001 (Calculus I)

- Monday, 01/24:

  From the homework assignment, we sketched the graph $y = f(x) = (1+x)^{1/x}$ using the TI-89. We saw that the limit as x approaches is the special number $e \doteq 2.7182818$. We discussed the domain of this function.

  We evaluated a bunch of limits from Sect. 2.3. The most important rule involves the canceling of the factor in the denominator. See Example 3 on p. 113.

  The most important examples looked like this:

  Suppose $f(x) = \sqrt{x}$. Then evaluate something which looks like this:

$$\lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{\sqrt{1+h} - \sqrt{1}}{h} = \lim_{h\to 0} \frac{\sqrt{1+h} - 1}{h}.$$

  We can simplify this by multiplying top and bottom by the algebraic conjugate: $\left(\sqrt{1+h}+1\right)$.

  We went ahead and stated the correspondence with the TANGENT LINE problem.

  This limit is EXACTLY equal to the slope of the tangent line to $y = f(x) = \sqrt{x}$ at the point where $x = 1$.

  Consider these Practice Problems: p. 118: #14, 15, 18.

  We listed all of the Limit Laws on pp. 110-112 and worked some examples.

  We discussed the weird behavior of $y = f(x) = \sin\left(\frac{1}{x}\right)$ when $x \to 0^+$. Thus,

$$\lim_{x\to 0^+} \sin\left(\frac{1}{x}\right) = d.n.e.$$

  But then we explored the SQUEEZE THEOREM for this limit:

$$\lim_{x\to 0^+} x \sin\left(\frac{1}{x}\right) = ???$$

  By graphing, we showed that

$$-x \le x \sin\left(\frac{1}{x}\right) \le x, \quad \text{for } x > 0.$$

  By the Squeeze Theorem, we have

$$\begin{aligned} \lim_{x\to 0^+} (-x) &\le \lim_{x\to 0^+} \left(x \sin\left(\frac{1}{x}\right)\right) \le \lim_{x\to 0^+} x \\ 0 &\le \lim_{x\to 0^+} \left(x \sin\left(\frac{1}{x}\right)\right) \le 0. \end{aligned}$$

  Thus, we have $\lim_{x\to 0^+} \left(x \sin\left(\frac{1}{x}\right)\right) = 0$.

  We mentioned the definition of continuity. More on this next time.