Bruce MacMillan's Test #3 (Practice Questions) – MATH 1401
Spring 2005

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Directions: Sorry, no notes, text, or technology for this portion of the test.

Show ALL algebra where appropriate.

(#1) If $y = (\ln(x))^{\tan(x)}$, then find y'.

(#2) Suppose $f(x) = 2x + \ln(x)$.

(a) Find the linear approximation of f at $x = 1$.

(b) Use that equation to approximate $f(1.001)$.

(#3) Alex has a helium balloon in his hand and Bonnie is standing 40 feet from him.

Alex lets the balloon go, and it rises vertically in the still air. A few seconds later, the balloon is 30 feet in the air and the instantaneous velocity of the balloon is +5 ft/sec upward.

(a) How fast is the distance between Bonnie and the balloon increasing when the balloon's height is $y = 30$ feet?

(b) How fast is the *angle of elevation* (angle between the ground and the line of sight of the balloon) measured from Bonnie's perspective increasing when the balloon's height is $y = 30$ feet? [What are the units of measure in the final answer?]

(#4) The height and radius of a right circular cylinder are equal, so the volume is $V = \pi h^3$. Suppose that the height (and radius) of the cylinder changes from 10 cm to 10.1 cm.

(a) Use differentials to approximate the change in the volume of the cylinder. (Find dV.)

(b) Write down the expression for the *relative* change in the volume.

(#5) Find two positive real numbers whose product is 45, and the sum of the first number plus three times the second number is a minimum.

(#6) A rectangular box with an open top and a square base is to be made out of 1200 cm^2 of material. Find the dimensions of the box that will *maximize* the volume of the box.

(#7) Find the *absolute* minimum and *absolute* maximum values of $f(x) = x^3 - 12x$ on the closed interval $[-3, 5]$.

(#8) Let $f(x) = x + \sin(2x)$.

We know that $x = \frac{\pi}{3}$ is a critical number (you can check this!) because $f'\left(\frac{\pi}{3}\right) = 0$.

Use the Second Derivative Test to determine if $\left(\frac{\pi}{3}, \frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)$ is relative minimum point, relative maximum point, or neither.

(#9) Given: $f(x) = \dfrac{4x}{(x+2)^2} \qquad f'(x) = \dfrac{8-4x}{(x+2)^3} \qquad f''(x) = \dfrac{8x-32}{(x+2)^4}$

(a) Write equations for all horizontal and vertical asymptotes of f.

(b) Find all (open) intervals where the f is decreasing.

(c) Find all (open) intervals where the f is increasing.

(d) Find all points (x, y) where f has local extrema (minima or maxima).

(e) Find all (open) intervals where f is concave down.

(f) Find all (open) intervals where f is concave up.

(g) Find all points (x, y) where f has inflection points.

(#10) Since $f(x) = \sqrt{2x+1}$ satisfies the hypothesis of the Mean Value Theorem on the interval $[0, 4]$, find ALL values of x in the interval $(0, 4)$ that satisfy the conclusion of the theorem.

(#11) Find each limit if it exists. Show all work.

(a) $\displaystyle\lim_{x \to 0} \frac{e^{3x} - 3x - 1}{x^2}$

(b) $\displaystyle\lim_{x \to +\infty} x * \sin\left(\frac{1}{x}\right)$

(c) $\displaystyle\lim_{x \to 0} (e^x + x)^{1/x}$

(#12) If $f(x) = 2\cos(x) - \dfrac{2}{\sqrt{x}}$, then find the general antiderivative of $f(x)$. [Find $F(x)$.]

(#13) Solve the differential equation: $f'(x) = x + \dfrac{1}{x} + \dfrac{1}{x^2}$ and $f(1) = 0$.