Activities Associated with Assignment #3 – MATH 1401
Spring 2005

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1. Each of these limits is equivalent to $f'(a)$.

   Determine the definition of $f(x)$ and the value of $x = a$.

   Example:
   $$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to 5} \frac{2^x - 32}{x - 5}.$$
   When we compare the definitions, we see that $a = 5$ [from the limit] and $f(x) = 2^x$ [from the substitution].

   (a) $\displaystyle\lim_{x \to e} \frac{\ln(x) - 1}{x - e}$

   (b) $\displaystyle\lim_{x \to 16} \frac{\frac{1}{\sqrt{x}} - \frac{1}{4}}{x - 16}$

   (c) TRICKY! $\displaystyle\lim_{x \to \pi/6} \frac{6\tan(x) - 2\sqrt{3}}{6x - \pi}$

2. Each of these limits is equivalent to $f'(a)$.

   Determine the definition of $f(x)$ and the value of $x = a$.

   Example:
   $$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{\cos(\pi + h) + 1}{h} = f'(\pi).$$
   We must have $a = \pi$, and from the substitution, we guess that $f(x) = \cos(x)$.

   We verify that $\cos(a) = \cos(\pi) = -1$, so $\cos(\pi + h) - \cos(\pi)$ is, in fact, $\cos(\pi + h) + 1$.✓

   (a) $\displaystyle\lim_{h \to 0} \frac{\sqrt[4]{16 + h} - 2}{h}$

   (b) $\displaystyle\lim_{h \to 0} \frac{(1 + h)^{10} - 1}{h}$

3. Now, instead of a, copy in x in its place. This gives us the actual derivative function.
   $$f'(x) = [f(x)]' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$
   Find $f'(x)$ if $f(x) = \dfrac{1}{\sqrt{x}}$.

Solutions

1. $x = a = ???\quad f(x) = ???$

   (a) $\lim_{x \to e} \dfrac{\ln(x) - 1}{x - e} \Rightarrow a = e$ and $f(x) = \ln(x)$. We verify that $\ln(e) = 1.\checkmark$

   (b) $\lim_{x \to 16} \dfrac{\frac{1}{\sqrt{x}} - \frac{1}{4}}{x - 16} \Rightarrow a = 16$ and $f(x) = \dfrac{1}{\sqrt{x}}$. We verify that $\dfrac{1}{\sqrt{16}} = \dfrac{1}{4}$.

   (c) Divide top and bottom by 6. $\lim_{x \to \pi/6} \dfrac{\frac{6\tan(x)}{6} - \frac{2\sqrt{3}}{6}}{\frac{6x}{6} - \frac{\pi}{6}} = \lim_{x \to \pi/6} \dfrac{\tan(x) - \frac{\sqrt{3}}{3}}{x - \frac{\pi}{6}} \Rightarrow$

   $a = \frac{\pi}{6}$ and $f(x) = \tan(x)$. We verify that $\tan\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}.\checkmark$

2. $x = a = ???\quad f(x) = ???$

   (a) $\lim_{h \to 0} \dfrac{\sqrt[4]{16 + h} - 2}{h} \Rightarrow a = 16$ and $f(x) = \sqrt[4]{x}$. We verify that $\sqrt[4]{16} = 2.\checkmark$

   (b) $\lim_{h \to 0} \dfrac{(1 + h)^{10} - 1}{h} \Rightarrow a = 1$ and $f(x) = x^{10}$. We verify that $1^{10} = 1.\checkmark$

3. $$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \lim_{h \to 0} \frac{\frac{\sqrt{x}}{\sqrt{x}\sqrt{x+h}} - \frac{\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h} = \lim_{h \to 0} \frac{\left(\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right)}{h} =$$

   $$\lim_{h \to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} = \lim_{h \to 0} \frac{\left(\sqrt{x} - \sqrt{x+h}\right)\left(\sqrt{x} + \sqrt{x+h}\right)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x} + \sqrt{x+h}\right)} = \lim_{h \to 0} \frac{x - (x+h)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x} + \sqrt{x+h}\right)} =$$

   $$\lim_{h \to 0} \frac{-h}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x} + \sqrt{x+h}\right)} = \lim_{h \to 0} \frac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x} + \sqrt{x+h}\right)} = \frac{-1}{\sqrt{x}\sqrt{x+0}\left(\sqrt{x} + \sqrt{x+0}\right)} =$$

   $$\frac{-1}{\sqrt{x}\sqrt{x}\left(2\sqrt{x}\right)} = -\frac{1}{2}\left(\frac{1}{x\sqrt{x}}\right) = -\frac{1}{2}x^{-3/2}.\checkmark$$

   This agrees with the Simple Power Rule. $\left[x^{-1/2}\right]' = -\dfrac{1}{2}x^{-3/2}$.

Calculus I BINGO!

Find 5 squares in a row (horizontally, vertically, or diagonally) which contain expressions equal to ZERO.
The center square is free. (Put an 'X' there!)

$\sin(0)$	$[28]'$	$\lim_{x\to 0} \lvert x\rvert$	$\cos\left(\frac{\pi}{2}\right)$	$\cos(0)$
$\lim_{x\to 2}\left(x^2-2x\right)$	$\lim_{x\to 1}(x-1)^{20}$	e^0	Slope of the line $y=23$	$x^2-\sqrt{x^4}$
$\left[x^2\right]'-2x$	0^0	$FREE!$ $FREE!$	$\ln(1)$	$\cos^{-1}(1)$
$\lim_{x\to 2^+}\left(\frac{2}{x-2}\right)$	$\lim_{x\to 0}\sin(3x)$	$\lim_{x\to 5}\sqrt{x^2-x-20}$	$[3x]'-3$	$\lim_{x\to 0}\left(x\sin\left(\frac{1}{x}\right)\right)$
0^1	$\sin^{-1}(0)$	$\tan(0)$	$\lim_{x\to 2}\left(\frac{x^2-4}{x^2+4}\right)$	Slope of the line $x=23$

Calculus I BINGO [Discussion]

1. The only winning line was:

			X	NO
		NO	X	
	NO		X	
NO			X	
			X	NO

2. Here are the items which were NOT equal to zero:

   (a) $\cos(0) = 1$.

   (b) 0^0 is *undefined.*

   (c) $e^0 = 1$.

   (d) Slope of the line $x = 23$ is *undefined.*

   (e) $\lim_{x \to 2^+} \frac{2}{x-2} = +\infty$.