RESEARCH STATEMENT FOR MICHAEL J. FERRARA

1. Introduction

My research interests fall into several categories, including extremal graph theory, graph labeling and graphs and graph operators. Here I will focus on 3 areas: potentially H-graphic degree sequences, graph subdivision and immersion problems and, lastly, cycle structure in graphs, which has been a growing part of my research as of late. While I do work on my own, I greatly enjoy collaborating on research projects, both with colleagues and as a key part of training graduate students.

2. Extremal Graph Theory

Extremal graph theory is the study of how graph invariants, such as minimum or maximum degree, connectivity, order and size can influence certain graph properties. The classic example of a theorem in this area is the renowned Turán theorem that states if G is a simple graph containing a sufficient number of edges, then G contains K_t as a subgraph[16].

2.1. **Potentially Graphic Degree Sequences.** A nonincreasing sequence of nonnegative integers $\pi = (d_1, d_2, \ldots, \; d_n)$ is said to be *graphic* if there is some graph G of order n having degree sequence π. In this case, we say that G *realizes* π and write $\pi = \pi(G)$.

Given a graphic sequence π and a graph H, we say that π is potentially H-graphic if there is some realization of π that has H as a subgraph. For a given H and nonnegative integer n, define $\sigma(H, n)$ to be the smallest integer m such that any n-term graphic sequence π with sum $\sigma(\pi) > m$ is potentially H-graphic. This is a natural variant of the Turán problem, and was first considered by Erdős, Jacobson and Lehel [2], where they demonstrated that $\sigma(K_3, n) = 2n - 2$ and conjectured that $\sigma(K_t, n) = (t-2)(2n-t+1)$. This conjecture was verified in several papers [12],[14], including for all $t \geq 6$ by Li, Song and Luo [13].

The complete, balanced multipartite graph K_s^t is comprised of t disjoint sets of s vertices each, such that each set is independent and each pair of vertices chosen from different sets is adjacent. In [5], Gould, Schmitt and I were able to determine $\sigma(K_2^t, n)$ and have given a lower bound on $\sigma(K_s^t, n)$ when $s \geq 3$. The bound given in [5] is confirmed in [1] using a new technique, the degree-stripping method. We have also used this method in [6] to determine $\sigma(F_k, n)$ where F_k denotes the friendship graph, which is k triangles intersecting at a vertex and in [4] to determine the potential number of an arbitrary union of disjoint cliques.

Additionally, in [11], for any graph H we construct a graphic sequence $\widehat{\pi}(H, n)$ such that $\sigma(\widehat{\pi}(H, n)) \leq \sigma(H, n) + 2$ for sufficiently large n. Also in [11] we show that equality holds for a large class of graphs, and we furthermore conjecture that equality holds. At this time, we do not know of any H for which the conjecture fails.

Recently, we have investigated potentially H-bigraphic sequences. A sequence $\pi = (s_1, \ldots, s_n, t_1, \ldots, t_m)$ is said to be bigraphic if there exists a bipartite graph $G = G(X \cup Y, E)$ such that $|X| = n$, and $|Y| = m$ with $(s_1, \ldots, s_n)$ and $(t_1, \ldots, t_m)$ being the degrees of the vertices in X and Y respectively. In [8], we determine the minimum degree sum that assures a bigraphic sequence has a realization containing a $K_{a,b}$, P_k or C_{2k}.

There are many interesting and difficult problems related to this field, and we are currently continuing our research in this area.

2.2. H-Linked Graphs.

A graph G is said to be *k-linked* if for any set of distinct, ordered vertices $Z = \{s_1, \ldots, s_k, t_1, \ldots, t_k\}$ there exist vertex disjoint paths $P_1, \ldots, P_k$ in G such that P_i is an $s_i - t_i$ path for all i. We have focused on generalizing the notion of a k-linked graph while considering constraints on minimum degree.

Let H be a multigraph, possibly containing loops. For any graph G, let $\mathcal{P}(G)$ denote the set of paths in G. An *H-subdivision* in G is a pair of mappings $f_1 : V(H) \to V(G)$ and $f_2 : E(H) \to \mathcal{P}(G)$ such that:
(i) f_1 is injective;
(ii) for every edge $xy \in E(H)$, $f_2(xy)$ is an $f_1(x) - f_1(y)$ path in G and distinct edges of H map to internally disjoint paths in G.

A graph G is *H-linked* if every injective map $f_1 : V(H) \to V(G)$ can be extended to an H-subdivision. Note that if we were to choose H to be a collection of k disjoint edges, then a graph G being H-linked is equivalent to G being k-linked. In [9] we were able to show that for sufficiently large n, if the minimum degree of G is at least $\frac{n+\eta(H)-2}{2}$ then G is H-linked. Here $\eta = \eta(H)$ is a parameter based on the structure of H.

Recently [10] we have extended the notion of an H-linked graph. Given G and H as above, an *H-imitation* in G is an injective map $f_1 : V(H) \to V(G)$ along with a function $f_2 : E(H) \to \mathcal{P}(G)$ such that distinct edges map only to edge-disjoint paths. In [10] we give minimum degree conditions for a graph G to contain an imitation of H with a maximum number of repeated vertices. It is important to note that the bounds in both [9] and [10] are sharp. Our work in this area is ongoing.

3. Cycle Structure in Graphs

A graph G is said to be *hamiltonian* if G contains a spanning cycle. The problem of determining when a graph has a hamiltonian cycle has been widely studied. In [10], we gave sharp conditions for a bipartite graph to contain k edge-disjoint hamiltonian cycles, extending the classic results in [15] and [3].

A *2-factor* in a graph G is a collection of disjoint cycles whose vertex set spans G. The problem of determining when a graph has a 2-factor is an extension of the hamiltonian problem, as a hamiltonian cycle is simply a specific type of 2-factor. Some of my recent work has focused on 2-factors in iterated line graphs. We recursively define the iterated line graphs of G by $L^1(G) = L(G)$ and for all $t \geq 2$, $L^t(G) = L(L^{t-1}(G))$. In [7] we give sufficient conditions for $L^k(G)$ to contain 2-factors of various types as well as characterizing those graphs G such that $L^k(G)$ has a 2-factor for all $k \geq 3$.

Along with Mike Jacobson at UCD, I am currently exploring several new problems pertaining to hamilonian graphs. I expect it to continue as a fruitful line of research.

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