

# Introduction to Graph Theory

## Section 1.4 Homework

### ”h”omework - Not collected

West: 2.1.7, 2.1.13, 2.1.18, 2.1.26, 2.1.34, 2.1.47

### ”H”omework - Collected

West: 2.1.63

and:

1) Let  $T$  be a tree and let  $n_i(T)$  denote the number of vertices of degree  $i$  in  $T$ .

a) Prove that  $n_1(T) = 2 + \sum_{i=2}^{\Delta(G)} (i-2)n_i(T)$ .

b) A tree  $T$  is a caterpillar if there is a path  $P$  in  $T$  such that every vertex is either on  $P$  or adjacent to a vertex on  $P$ . Use part (a) to show that if  $\pi$  is the degree sequence of a tree, then  $\pi$  has a realization that is a caterpillar.

2) Let  $G$  be a  $k$ -regular graph for some  $k \geq 3$ . Prove that any two of the following conditions imply the third.

1.  $G$  has diameter  $d$ ,
2.  $G$  has girth  $2d + 1$ , or
3.  $G$  has order  $1 + k \frac{(k-1)^d - 1}{k-2}$ .

3) Let  $G$  be a order  $n$  with diameter 2 and maximum degree  $k$ . Prove that if  $k \in \{n-3, n-4\}$  then  $G$  has at least  $2n-5$  edges. Show as well that this result is best possible.