7.6

(6) $f_{ave} = \int_0^{\pi/3} \frac{1}{\pi/3 - 0} \sec x \tan x \, dx$

$= \frac{3}{\pi} \int_0^{\pi/3} \sec x \tan x \, dx$

$= \frac{3}{\pi} \left[\sec x \right]_0^{\pi/3}$

$= \frac{3}{\pi} \left[\sec \frac{\pi}{3} - \sec 0 \right]$

$= \frac{3}{\pi} \left[2 - 1 \right]$

$= \frac{3}{\pi}$

(8) $f_{ave} = \frac{1}{\ln 5 - (-1)} \int_{-1}^{\ln 5} e^x \, dx$

$= \frac{1}{\ln 5 + 1} \left[e^x \right]_{-1}^{\ln 5}$

$= \frac{1}{\ln 5 + 1} \left[e^{\ln 5} - e^{-1} \right]$

$= \frac{1}{\ln 5 + 1} \left[5 - \frac{1}{e} \right]$

$= \frac{1}{\ln 5 + 1} \left[\frac{5e - 1}{e} \right]$

$= \frac{5e - 1}{e(\ln 5 + 1)}$

(22) (a) $a_{ave} = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} a(t) \, dt$

$a_{ave} = \frac{1}{5 - 0} \int_0^5 (t + 1) \, dt$

$= \frac{1}{5} \left[\frac{t^2}{2} + t \right]_0^5$

(22) continued...

$= \frac{1}{5} \left[\left(\frac{25}{2} + 5 \right) - (0) \right]$

$= \frac{7}{2}$

(b) $a_{ave} = \frac{1}{\frac{\pi}{4} - 0} \int_0^{\pi/4} a(t) \, dt$

$= \frac{1}{\pi/4} \left[v(t) \right]_0^{\pi/4}$

$= \frac{4}{\pi} \left[\cos t \right]_0^{\pi/4}$

$= \frac{4}{\pi} \left[\cos \frac{\pi}{4} - \cos 0 \right]$

$= \frac{\pi}{4} \left[\frac{\sqrt{2}}{2} - 1 \right]$

$= \frac{\sqrt{2}\,\pi}{8} - \frac{\pi}{4}$

$= \frac{(\sqrt{2} - 2)\pi}{8}$

7.7

(2) $W = \int_0^5 F(x) \, dx$

$= \int_0^2 F(x) \, dx + \int_2^5 F(x) \, dx$

Geometry

$= (2)(40) + \frac{1}{2}(3)(40)$

$= 80 + 60$

$= 140 \text{ Nm}$ or 140 J

$\boxed{7-7}$

(6) $F = 40\,N$ is constant.

Work = Force • Distance

$$W = 40 \cdot \int_0^{15} v(t)\,dt$$

Geometry

$$= 40 \cdot \left(\tfrac{1}{2}\right)(15)(10)$$

$$= 3000 \text{ J}$$

Note: I think that the author probably meant for velocity to be m/s not ft/s.

(8) (a) $F = kx$

$$45 = k \cdot 0.05$$

$$900 = k \Rightarrow F(x) = 900x$$

(b) $W = \int_a^b F(x)\,dx$

$$W = \int_0^{.03} 900x\,dx$$

$$= 450x^2\Big]_0^{.03}$$

$$= 450(.03)^2$$

$$= 0.405 \text{ J}$$

(c) $W = \int_{.05}^{.1} 900x\,dx$

$$= 450x^2\Big]_{.05}^{.1}$$

$$= 450(.1)^2 - 450(.05)^2$$

$$= 3.375 \text{ J}$$

(10) $F = kx$

$$6 = k\left(\tfrac{1}{2}\right)$$

$$12 = k \Rightarrow F(x) = 12x$$

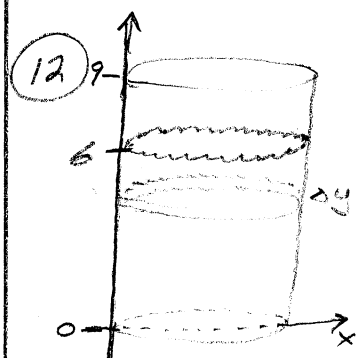
$$W = \int_0^2 12x\,dx$$

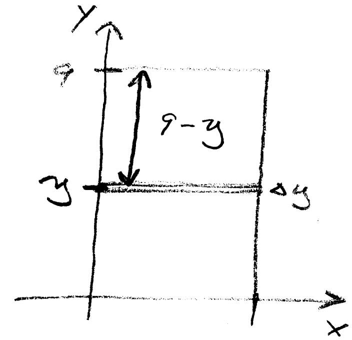
$$= 6x^2\Big]_0^2$$

$$= 6(4) - 0$$

$$= 24 \text{ J}$$

(12)





$$W = \int_a^b (\text{weight of disc})(\text{distance})\,dy$$

The distance to lift a disc of water is: distance $= 9 - y$

The weight of a disc is (Volume of disc)(weight density water)

$$= (\pi r^2 h)(62.4)$$

$$= (\pi \cdot 5^2 \cdot \Delta y)(62.4)$$

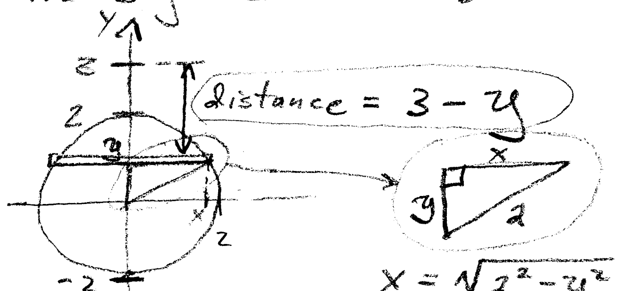
$$= 1560\pi\,\Delta y$$

$$W = \int_0^6 1560\pi(9 - y)\,dy$$

$$= 1560\pi\left[9y - \frac{y^2}{2}\right]_0^6$$

$$= 1560\pi[54 - 18] = 56160\pi \text{ ft·lb}$$

(16) The tank is on its side in figure Ex-16 !



$x = \sqrt{2^2 - y^2}$

$x = \sqrt{4 - y^2}$

Weight of thin slab of liquid :

Weight = (Volume of slab)(50 lb/ft³)

$= \left(2\sqrt{4-y^2}\right)(10)(50)$

$= 1000\sqrt{4-y^2}$

$$\text{Work} = \int_{-2}^{2} (\text{weight})(\text{distance})\,dy$$

$$= \int_{-2}^{2} 1000\sqrt{4-y^2}\,(3-y)\,dy$$

$$= \int_{-2}^{2} \left(3000\sqrt{4-y^2} - 1000\,y\sqrt{4-y^2}\right)dy$$

$$= 3000\underbrace{\int_{-2}^{2}\sqrt{4-y^2}\,dy}_{\mathrm{I}} - 1000\underbrace{\int_{-2}^{2} y\sqrt{4-y^2}\,dy}_{\mathrm{II}}$$

(I) This is the area of a half-circle of radius 2 which is

$$\tfrac{1}{2}(\pi \cdot 2^2) = 2\pi$$

(II) u-substitution :

Let $u = 4 - y^2$ $\qquad y = 2 \Rightarrow u = 4 - 2^2 = 0$

$\frac{du}{dy} = -2y$ $\qquad y = -2 \Rightarrow u = 4 - (-2)^2 = 0$

$\frac{du}{-2} = y\,dy$

So $\int_{-2}^{2} y\sqrt{4-y^2}\,dy = -\frac{1}{2}\int_0^0 \sqrt{u}\,du$

$= 0$ (why?)

Thus

$W = 3000(\mathrm{I}) - 1000(\mathrm{II})$

$= 3000(2\pi) - 1000(0)$

$= 6000\pi$ ft·lbs

$\boxed{8.1}$

(4) Let $u = x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$$\int 4\tan(x^2)\,x\,dx$$

$$= \int 4\tan(u)\,\frac{du}{2}$$

$$= 2\int \tan(u)\,du$$

Formula #13 page 511

$$= -2\ln|\cos u| + C$$

$$= -2\ln|\cos(x^2)| + C$$

(6) $\int \frac{1}{9+4x^2}\,dx = \int \frac{1}{3^2+(2x)^2}\,dx$

Let $u = 2x$

$\frac{du}{dx} = 2$

$\frac{du}{2} = dx$

$$= \int \frac{1}{3^2+u^2}\cdot\frac{du}{2}$$

$$= \frac{1}{2}\cdot\frac{1}{3}\tan^{-1}\left(\frac{u}{3}\right) + C$$

Using formula 22 page 511

$$= \frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right) + C$$

$\boxed{8.1}$

(10) $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\,dx = \int \frac{x}{\sqrt{1-(x^2)^2}}\,dx$

Let $u = x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$$= \int \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{2}$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du$$

You should know (memorize) this integral

$$= \frac{1}{2}\sin^{-1}(u) + C$$

$$= \frac{1}{2}\sin^{-1}(x^2) + C$$

(18) $\displaystyle\int \frac{dx}{x(\ln x)^2} = \int \frac{1}{(\ln x)^2}\cdot\frac{dx}{x}$

Let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{dx}{x}$

$$= \int \frac{1}{u^2}\,du$$

$$= \int u^{-2}\,du$$

$$= -u^{-1} + C$$

$$= -\frac{1}{u} + C$$

$$= -\frac{1}{\ln x} + C$$

(24) $\displaystyle\int \frac{\cos(\ln x)}{x}\,dx = \int \cos(\ln x)\cdot\frac{dx}{x}$

Let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{dx}{x}$

$$= \int \cos(u)\,du$$

$$= \sin(u) + C$$

$$= \sin(\ln x) + C$$

(28)

$\displaystyle\int \frac{e^x}{\sqrt{4-e^{2x}}}\,dx = \int \frac{e^x}{\sqrt{4-(e^x)^2}}\,dx$

Let $u = e^x$

$\frac{du}{dx} = e^x$

$du = e^x dx$

$$= \int \frac{1}{\sqrt{4-(e^x)^2}}\cdot e^x dx$$

$$= \int \frac{1}{\sqrt{4-u^2}}\,du$$

$$= \int \frac{1}{\sqrt{2^2-u^2}}\,du$$

Formula #21

$a = 2$

$$= \sin^{-1}\left(\frac{u}{2}\right) + C$$

$$= \sin^{-1}\left(\frac{e^x}{2}\right) + C$$