Material for Test #2 – MATH 2411

Directions:

1. You may use the note sheet provided.
2. No technology whatsoever.
3. You must complete the test in 1 hour and 50 minutes.
4. Show all work on free response questions. The numbers used in the calculations are relatively small. No decimal approximations to square roots are necessary.
5. You need not show any work for multiple-choice, matching, and True/False questions.
6. Box/circle/highlight your final answers.
7. You may use the back of the sheets as scratch paper, but please indicate clearly where your work is located for each problem.

At a bare minimum, you are responsible for solving the following types of questions:

(#1) If we give you this reduction formula:

$$\int \cos^n(x)\,dx = \frac{1}{n}\cos^{n-1}(x)\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)\,dx,$$

then evaluate this antiderivative:

$$\int \cos^4(\theta)\,d\theta = ???$$

(#2) Evaluate:

(a)

$$\int_1^e x^4 \ln(x)\,dx = ???$$

(b)

$$\int \sin^{-1}(2x)\,dx$$

(c)

$$\int x\sec^2(x)\,dx$$

(#3) Suppose we have the region trapped beneath $y = \sin(x)$ over $[0, \pi]$, above the x-axis.

We create a solid of revolution whose volume can be evaluated by the Disk Method. Find the volume.

(#4) Evaluate:

(a)

$$\int \cos^2(3x)\sin^3(3x)\,dx = ???$$

(b)

$$\int_0^{\pi/4} \tan(\theta)\sec^4(\theta)\,d\theta = ???$$

(#5) Evaluate:

(a)

$$\int \frac{dx}{x^2\sqrt{1-x^2}} = ???$$

(b) Hint: You'll need $\int \sec(\theta)\,d\theta$.

$$\int \frac{x^2}{(x^2+1)^{3/2}}dx = ???$$

(#6) Evaluate:

$$\int \frac{11x+17}{2x^2+7x-4}dx = ???$$

(#7) Find the partial fraction decomposition for:

$$\frac{3x^2+3x+1}{x(x+1)^2} = \frac{???}{???} + \frac{???}{???} + \frac{???}{???}$$

No integration necessary.

(#8) Evaluate. If the integral diverges, then show why this is true.

(a)

$$\int_0^1 \frac{1}{\sqrt[3]{x}}dx = ???$$

(b)

$$\int_0^{+\infty} \frac{e^{-t}}{1+e^{-2t}}dt = ???$$

(c)

$$\int_{-\infty}^{+\infty} x\,dx = ???$$

(#9) Separable. Solve the IVP.

$$y' = y^2 * \sqrt{x},\ y(1) = 3.$$

(#10) Integrating factor. Solve for the general solution (one constant of integration).

$$y' + 3y = x^2$$

Hint: Tabular integration will help here.

(#11) Complete Problem #7 on p. 608. [Radon-222]

(#12) A tank initially contains 50 gal of pure water. Then at time $t = 0$ brine containing 2 lb/gal of salt is pumped into the tank at a rate of 3 gal/min and the mixed solution is drained from the tank at the same rate.

(a) Find an initial value problem whose solution is $y(t)$ that models the problem.
(b) Solve your initial value problem from part (a) to find a formula for $y(t)$.

Formula Sheet for Test #2 – Math 2411

You need to memorize integration formulas #1 – 5, 7 – 12, 21 and 22 on page 511. The following will be provided:

$\int b^u\,du = \dfrac{b^u}{\ln b} + C \qquad (b>0, b\neq 1)$	$\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C$
$\int \tan u\,du = -\ln\lvert\cos u\rvert + C$	$\int \cot u\,du = \ln\lvert\sin u\rvert + C$
$\int \sec u\,du = \ln\lvert\sec u + \tan u\rvert + C$	

You need to know your Right Triangle Trigonometry very well. You also need to know (have memorized) the Fundamental Identities such as $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\sec\theta = \dfrac{1}{\cos\theta}$ as well as the Pythagorean identities like $\sin^2\theta + \cos^2\theta = 1$ and $\tan^2\theta + 1 = \sec^2\theta$. The following trigonometric identities will be provided:

Double-Angle Formulas:	**Half-Angle Formulas:**
$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $= 2\cos^2\theta - 1$ $= 1 - 2\sin^2\theta$	$\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$ $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$