

NAME:

MIDTERM

Open books, notes, homeworks. You may use calculators for the basic four operations (+, -, *, /). Gessed answers are NOT accepted. Good luck!

(20 pt) 1) Construct a quadrature rule of the form:

$$\int_{-2}^2 f(x) dx \approx Af(-1) + Bf(0) + Cf(1)$$

that is exact for the highest possible degree polynomial. For which degree of polynomial is the formula you derived exact?

(20 pt) 2) The family of orthogonal polynomials with respect to the inner product given by

$$\langle f, g \rangle = \int_0^1 fg \, dx$$

is the Legendre polynomial evaluated at $2x - 1$, i.e., $p_0(x) = 1$, $p_1(x) = 2x - 1$, $p_2(x) = 4x^2 - 4x + 2/3$, ... Find the two-point Gaussian quadrature rule to approximate

$$\int_0^1 g \, dx \approx Ag(x_0) + Bg(x_1)$$

and determine the highest degree of the polynomial which is exactly integrated by this rule.

3) We wish to approximate the initial value problem

$$\begin{aligned}x' &= f(t, x) \\ x(0) &= x_0\end{aligned}$$

with the following numerical method

$$x_{n+1} = x_{n-1} + 2hf(t_n, x_n)$$

where $t_i = t_0 + ih$, i.e., the nodes are equally spaced points.

(10 pt) a) Classify the method

(10 pt) b) Is the method consistent? Justify your answer.

(10 pt) c) Is the method stable? Justify your answer.

(10 pt) d) Is the method A -stable? Justify your answer.