

NAME: _____

MIDTERM

Open books, notes, homeworks. You may use calculators or notebooks. Gussed answers are NOT accepted. Good luck!

(20 pt) 1) a) What are the condition numbers of $\cos(x)$ at $0, \pi/4$ and $\pi/2$?

b) If we perturb $x = \pi/4$ by 0.01 what is the relative size of the perturbation on $\cos(x)$?

(20 pt) 2) a) Formulate the Newton's method to solve the equation

$$x = e^x - 5$$

b) If we use an initial guess $x^{(0)} = 100$, will the Newton method converge? Justify without using the method to compute its successive iterates.

c) Consider now the iterative method given by:

$$x^{(k+1)} = e^{x^{(k)}} - 5$$

Will this method converge for $x^{(0)} = 100$? Justify without using the method to compute its successive iterates.

(20 pt) 3) Compute the condition number using the $\|\cdot\|_\infty$ matrix norm for

$$A = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1/2 & 0 \\ -1 & -2 & 10 \end{pmatrix}$$

(20 pt) 4) Let us consider the problem of solving $Ax = b$ using iterative methods. If we partition our matrix A into

$$A = D - C_L - C_U$$

then the Jacobi method can be written as

$$x^{(k)} = G_J x^{(k-1)} + D^{-1}b$$

where

$$G_J = D^{-1}(C_L + C_U).$$

Now consider the *weighted Jacobi method* given by

$$x^{(k)} = [(1 - w)I + wG_J]x^{(k-1)} + wD^{-1}b.$$

For the matrix

$$A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

will the weighted Jacobi method converge for $w = 3/2$ for any initial guess? Justify your answer using the theory in Section 4.6.