Math 5660 (Numerical Analysis I) **Fall 1995**

NAME: ______________________________

MIDTERM

Open books, notes, homeworks. You may use calculators or notebooks. Guessed answers are NOT accepted. Good luck!

(10 pt) 1) A quadratic equation $ax^2 + bx + c = 0$ has roots given by:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{1}$$

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{2}$$

Find the roots of:

$$x^2 - 890000x + 1 = 0 \tag{3}$$

using equations (1) and (2) in a floating decimal point machine with 8 digits in the mantissa using rounding in the approximation of numbers in each calculation. If you see problems with your answers, then explain what they are and recompute the roots with an alternative procedure you may consider more suitable using the same machine.

(15 pt) 2) a) Find the series corresponding to

$$\int_0^{0.1} \ln(1+x)\, d\, x$$

by using a power series expansion for the integrand. How many terms do you need to sum from this series to get the value of the integral with an absolute error less than 10^{-4}? What is the value of the integral within 10^{-4} of absolute error?

b) Find the series corresponding to

$$\int_0^{1.0} \ln(1+x)\, d\, x$$

by using a power series expansion for the integrand. How many terms do you need to sum from this series to get the value of the integral with an absolute error less than 10^{-4}? (Do not compute the value of the integral in this case)

(25 pt) 3) a) Find the linear polynomial ($f_l^*(x)$) that approximates $f(x) = \sin(\pi x)$ on $[-1, 1]$ using the least-squares method with respect to the weighting function $w(x) = 1$ on $[-1, 1]$.

b) Sketch the McLaurin series for $\sin(\pi x)$ including up to the linear term, the expression obtained in the previous item using the least-squares method, and the function $\sin(\pi x)$ on $[-1, 1]$. What are the errors using each approximation at $x = 5/6$?

c) Find the quadratic polynomial ($f_q^*(x)$) that approximates $f(x) = \sin(\pi x)$ on $[-1, 1]$ using the least-squares method with respect to the weighting function $w(x) = 1$ on $[-1, 1]$.