Math 5660 (Numerical Analysis I) **Fall 1994**

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Open books, notes, homeworks. You may use calculators or notebooks. Guessed answers are NOT accepted. Good luck!

(10 pt) 1) Determine the maximum error for

$$y = x_1^2 + x_2 x_3$$

where $x_1 = 1.0 \pm 0.1, x_2 = 3.0 \pm 0.1, x_3 = 2.0 \pm 0.1$.

(10 pt) 2) Evaluate the following integral with an absolute error less than 10^{-7}

$$\int_{10}^{\infty} \frac{1}{x^4 + x^2}\, d\, x$$

Note: You should use an appropriate series expansion of the integrand valid on $(10, \infty)$.

3) Consider the first three *Walsh* functions given by

$$W_0(x) = 1 \quad -1 \le x \le 1$$

$$W_1(x) = \begin{cases} 1 & -1 \le x < 0 \\ -1 & 0 \le x \le 1 \end{cases}$$

$$W_2(x) = \begin{cases} 1 & -1 \le x < -1/2 \\ -1 & -1/2 \le x < 1/2 \\ 1 & 1/2 \le x < 1 \end{cases}$$

(10 pt) a) Show that the set $S = \{W_0, W_1, W_2\}$ form an orthogonal set of functions on $[-1, 1]$ with respect to the Euclidean (or L_2-) norm.

(10 pt) b) Find the least-squares approximation to $f(x) = x$ on $[-1, 1]$ with respect to the L_2-norm using the set S from item a). Then sketch your approximation compared to $f(x) = x$ on $[-1, 1]$.

(10 pt) c) Consider now the set of functions $V = \{W_0, W_1, W_2, W_3\}$ where W_0, W_1, W_2 are given in item a) and

$$W_3(x) = \begin{cases} 1 & -1 \leq x < -1/2 \\ -1 & -1/2 \leq x < 0 \\ 1 & 0 \leq x < 1/2 \\ -1 & 1/2 \leq x \leq 1 \end{cases}$$

Based on the fact that this additional function W_3 is orthogonal to the set S given in item a) (You don't need to check this), compute the least-squares approximation to $f(x) = x$ on $[-1, 1]$ with respect to the L_2-norm using the set V.